Pendulum carousel

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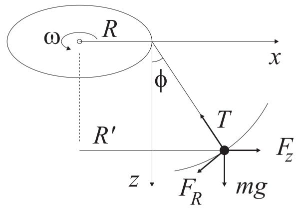
Solution: *The retroactive force is,*

$$F_R = -mg\sin\phi + \frac{mv^2}{R'}\cos\phi \ .$$

We have $v(x) = (R+x)\omega = (R + l\sin\phi)\omega$ *and* $R' = R + x$. *Hece,*



$$F_R = -mg\sin\phi + m\omega^2(R + l\sin\phi)\cos\phi \ .$$

The equation of motion, hence, is,

$$m\ddot{s} = -mg\sin\phi + m\omega^2(R + l\sin\phi)\cos\phi \ .$$

Since $s = l\phi$, $\ddot{s} = l\ddot{\phi}$, *follows* $l\ddot{\phi} = -g\sin\phi + \omega^2(R + l\sin\phi)\cos\phi$ *or,*

$$\ddot{\phi} + \frac{g}{l}\sin\phi - \omega^2(\frac{R}{l} + \sin\phi)\cos\phi = 0 \ .$$

For small displacements we have, $\cos\phi \approx 1$ *e* $\sin\phi \approx \phi$, *d.h.*

$$\ddot{\phi} + \left(\frac{g}{l} - \omega^2\right) - \omega^2\frac{R}{l} = 0 \ .$$

The solution of the homogeneous differential equation is,

$$\phi_h = \sin\sqrt{\frac{g}{l} - \omega^2 t}$$

A particular solution of the inhomogeneous differential equation is,

$$\phi_i = \frac{\omega^2(R/l)}{g/l - \omega^2}$$

The total solution is $\phi = \phi_h + \phi_i$. *The oscillation period is,*

$$T = \frac{2\pi}{\sqrt{g/l - \omega^2}}$$

For large angular velocities, $\omega = \sqrt{g/l}$, *the period of oscillation tends to infinity because the centrifugal force dominates.*