U-shaped water tube

Philippe W. Courteille, 05/02/2021

Solution: *If A is the cross section, the total mass of the water is $M = \rho V = \rho AL$, where ρ is the density of water. When the column moves at velocity v, it has kinetic energy,*

$$E_{kin} = \frac{M}{2} v^2 \ .$$

To put the mass $M_z = \rho V_z = \rho Az$ above the resting level of the water, such that the center of mass is a distance z above the low level, we need to provide the work,

$$-W = gM_z z = g\rho Az^2 = E_{pot} \ .$$

Energy conservation requires,

$$E = E_{kin} + E_{pot} = \frac{\rho AL}{2}\dot{z}^2 + g\rho Az^2 = \frac{M}{2}\dot{z}^2 + \frac{2gM}{2L}z^2 = const.$$

Hence, comparing the coefficient of the potential energy with $\frac{M}{2}\omega_0^2$,

$$\omega_0 = \sqrt{\frac{2g}{L}} \ .$$