Fourier expansion of sea waves

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Solution: *The coefficients of expansion are,*

$$a_0 = \tfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)d(kx) = \tfrac{1}{\pi}\int_{-\pi}^{\pi} (kx)^2 d(kx) = \frac{u^3}{3\pi}\Big|_{-\pi}^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \tfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nkx d(kx) = \tfrac{1}{\pi}\int_{-\pi}^{\pi} (kx)^2 \cos nkx d(kx)$$

$$= \tfrac{1}{n^3}[(n^2z^2-2)\sin nz + 2nz\cos nz]\big|_{-\pi}^{\pi} = \tfrac{4\cos n\pi}{n^2}$$

$$b_n = \tfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nkx d(kx) = 0$$

$$f(x) = \tfrac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nkx + b_n \sin nkx) = \tfrac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\tfrac{(-1)^n}{n^2}\cos nkx\ .$$

Note that the function can be parametrized as $f(x) = (mod(kx-\pi, 2\pi)-\pi)^2$ *for numerical treatment.*

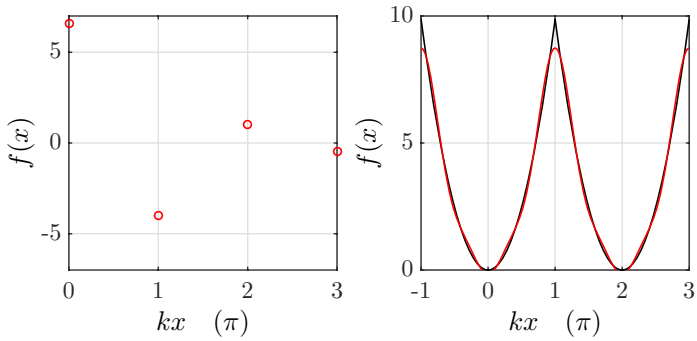

Figure 2.23: (code)