Fourier expansion of a rectified signal

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Solution: *The expansion coefficients are,*

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)d(\omega t) = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos\frac{\omega t}{2} d(\omega t) = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2} \cos u du = \frac{2}{\pi}\sin u\Big|_{-\pi/2}^{\pi/2} = \frac{4}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos\omega t d(\omega t) = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos\frac{\omega t}{2}\cos nkt d(\omega t) = \frac{1}{\pi}\int_{-\pi}^{\pi}\cos\frac{z}{2}\cos nz dz = -\frac{4}{\pi}\frac{\cos\pi n}{4n^2-1}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin n\omega t d(\omega t) = 0$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos n\omega t + b_n \sin n\omega t) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}\cos n\omega t \ .$$

The graph on the left of the figure shows Fourier components, while the graph on the right shows the original function $f(t)$ as well as the expanded function up to third order.

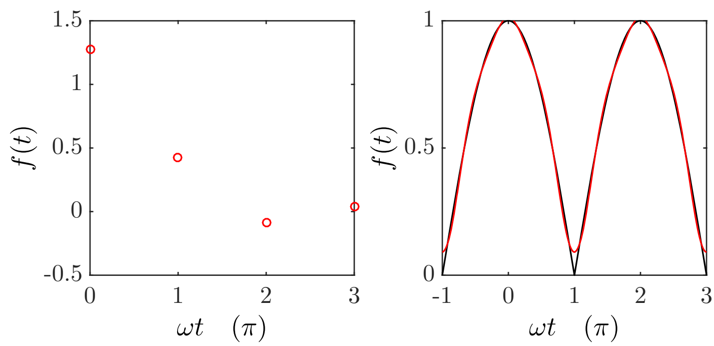

Figure 2.24: (code)