Action of a low pass filter on a spectrum

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Solution: *For the function $S(t)$ we calculate the coefficients,*

$$a_0 = \frac{\omega}{\pi}\int_0^{2\pi/\omega}\frac{\sin\omega t}{|\sin\omega t|}dt = 0 \quad , \quad a_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega}\frac{\sin\omega t}{|\sin\omega t|}\cos n\omega t dt = 0 \quad ,$$

$$b_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega}\frac{\sin\omega t}{|\sin\omega t|}\sin n\omega t dt = \frac{2}{\pi}\int_0^{\pi}\sin n\alpha d\alpha = -\frac{2}{\pi}\frac{\cos\pi n - 1}{n} = \frac{2}{n\pi}[1-(-1)^n] \ ,$$

with the consequence,

$$S(t) = \frac{2}{\pi}\sum_{n=1,3,\ldots}\frac{\sin n\omega t}{n} \ .$$

Graph (a) shows the amplitudes b_n and graph (b) the approximation by the Fourier series to orders 1, 2, and 20 (red). Attenuating the amplitudes as,

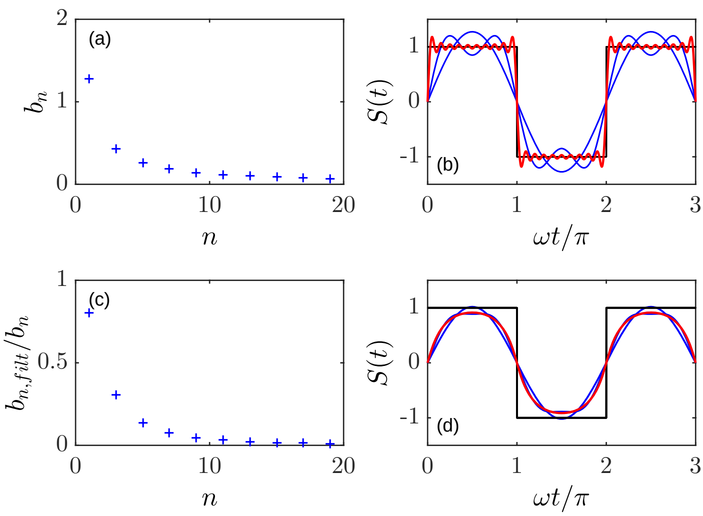

Figure 2.25: (code)

$$\tilde{b}_n = \frac{b_n}{1+(n\omega/\omega_g)^2} \ ,$$

we have,

$$S(t) = \frac{2}{\pi}\sum_{n=1,3,\ldots}\frac{1}{n}\frac{\sin n\omega t}{1+(n\omega/\omega_g)^2} \ .$$

Graph (c) shows the amplitudes attenuated by the filter $\tilde{b}_n$ and graph (d) the Fourier series approximation to orders 1, 2 and 20 (red). Obviously, the filtered signal is *almost sinusoidal. A higher order filter can improve the result. Calculating the harmonic distortion of the numerically filtered rectangular signal gives* $k \approx 0.1466$. *For a second order filter of type* $1/\left(1+(\omega/\omega_g)^4\right)$, *we obtain* $k \approx 0.0018$.