

Green's function

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Solution: We have,

$$\begin{aligned} \int d\theta e^{i(kx\theta - kx'\theta - kz\theta^2/2)} &= \int d\theta e^{ik[(x-x')\theta - z\theta^2/2]} = \int d\theta e^{ik\left[\frac{(x-x')^2}{2z} - \frac{z}{2}\left(\theta - \frac{x-x'}{z}\right)^2\right]} \\ &= e^{ik\frac{(x-x')^2}{2z}} \int d\theta' e^{-\frac{ikz}{2}\theta'^2} = e^{ik\frac{(x-x')^2}{2z}} \sqrt{\frac{2}{ikz}} \int d\tilde{\theta} e^{-\tilde{\theta}^2} = e^{ik\frac{(x-x')^2}{2z}} \sqrt{\frac{2\pi}{ikz}}. \end{aligned}$$