Propagating standing wave

Philippe W. Courteille, 05/02/2021

Solution: *a. We have,*

$$\begin{aligned}|e^{\imath(k_+x-\omega_+t)}+e^{\imath(-k_-x-\omega_-t)}| &= |e^{\imath\omega_+(x/c-t)}+e^{\imath\omega_-(-x/c-t)}|\\ &= \sqrt{2+2\cos[\omega_+(x/c-t)-\omega_-(-x/c-t)]}\\ &= 2\cos\left[\tfrac{\omega_++\omega_-}{2c}x-\tfrac{\omega_+-\omega_-}{2}t\right]\simeq 2\cos\left[kx-\tfrac{\omega_+-\omega_-}{2}t\right]\ .\end{aligned}$$

In real notation [7] *defining*

$$a+b=k_+x-\omega_+t\qquad,\qquad a-b=-k_-x-\omega_-t\ ,$$

or

$$a=\tfrac{k_+-k_-}{2}x-\tfrac{\omega_++\omega_-}{2}t\qquad,\qquad b=\tfrac{k_++k_-}{2}x-\tfrac{\omega_+-\omega_-}{2}t$$

we can write,

$$\begin{aligned}\cos(k_+x-\omega_+t)+\cos(-k_-x-\omega_-t) &= \cos(a+b)+\cos(a-b)=2\cos a\cos b\\ &= 2\cos\left(\tfrac{k_+-k_-}{2}x-\tfrac{\omega_++\omega_-}{2}t\right)\cos\left(\tfrac{k_++k_-}{2}x-\tfrac{\omega_+-\omega_-}{2}t\right)\\ &\simeq 2\cos\left(\tfrac{k_+-k_-}{2}x-\omega t\right)\cos\left(kx-\tfrac{\omega_+-\omega_-}{2}t\right)\ .\end{aligned}$$

b. Interference produces a standing wave that moves slowly with the group velocity, $v_g=\frac{\omega_1-\omega_2}{k}$.

[7] We note,

$$\begin{aligned}|z_1+z_2|^2=|z_1|^2+|z_2|^2+z_1z_2^*+z_1^*z_2=x_1^2+y_1^2+x_2^2+y_2^2+2x_1x_2+2y_1y_2=(x_1+x_2)^2+(y_1+y_2)^2\\ =|\mathrm{Re}z_1+\mathrm{Re}z_2|^2+|\mathrm{Im}z_1+\mathrm{Im}z_2|^2\ .\end{aligned}$$