Spherical waves

Philippe W. Courteille, 05/02/2021

Solution: *We proceed in Cartesian coordinates. On one hand, we have,*

$$\frac{1}{c^2}\frac{d^2}{dt^2}\frac{\sin(kr-\omega t)}{kr} = -\frac{\omega^2}{c^2}\frac{\sin}{kr} \ .$$

On the othe hand,

$$\begin{aligned}\frac{d^2}{dx^2}\frac{\sin(kr-\omega t)}{kr} &= \frac{d^2}{dx^2}\frac{\sin(k\sqrt{x^2+y^2+z^2}-\omega t)}{k\sqrt{x^2+y^2+z^2}} \\ &= \frac{k^2r^2x^2\sin -3x^2\sin +r^2\sin -3kx^2r\cos +kr^3\cos}{kr^5} \ ,\end{aligned}$$

such that,

$$\nabla^2\frac{\sin(kr-\omega t)}{kr} = \frac{k^2r^4\sin -3r^2\sin +3r^2\sin -3kr^3\cos +3kr^3\cos}{kr^5} = \frac{k^2\sin}{kr} \ .$$

In spherical coordinates the task is trivial.