Interference in spherical waves

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Solution: *Destructive interference occurs when*

$$\cos[k|\mathbf{r}-\mathbf{r}_+|-\omega t]+\cos[k|\mathbf{r}-\mathbf{r}_-|-\omega t]$$
$$=2\cos\frac{k|\mathbf{r}-\mathbf{r}_+|+k|\mathbf{r}-\mathbf{r}_-|-2\omega t}{2}\cos\frac{k|\mathbf{r}-\mathbf{r}_+|-k|\mathbf{r}-\mathbf{r}_-|}{2}=0$$

at all times. This requires,

$$|\mathbf{r}-\mathbf{r}_+|-|\mathbf{r}-\mathbf{r}_-|=\left(n+\tfrac{1}{2}\right)\lambda\ ,$$

for integer n. This means that for a distant observation point, $r\gg R$,

$$\left(n+\tfrac{1}{2}\right)\lambda=\sqrt{r^2+R^2-2R\mathbf{r}\hat{\mathbf{e}}_z}-\sqrt{r^2+R^2+2R\mathbf{r}\hat{\mathbf{e}}_z}$$
$$=r\sqrt{1+\frac{R^2}{r^2}-\frac{2Rz}{r^2}}-r\sqrt{1+\frac{R^2}{r^2}+\frac{2Rz}{r^2}}\simeq-2R\cos\theta\ .$$

Constructive interference lines are obtained similarly.