Sonic Doppler effect

Philippe W. Courteille, 05/02/2021

Solution: *Be x_{fnt} the position of the sound source and $v_{fnt} > 0$ its speed relative to the x-axis. The wavelength produced by the source in a medium is given by,*

$$\lambda_{med} = \lambda_{fnt}\left(1 - \frac{v_{fnt}}{c}\right) = \frac{c}{f_{fnt}}\left(1 - \frac{v_{fnt}}{c}\right) \ ,$$

where $c > 0$ in the region $x > x_{fnt}$ and $c < 0$ in the region $x < x_{fnt}$. Now, an observer in the region $x > x_{fnt}$ that moves with speed v_{obs} relative to the x-axis probes this wavelength with frequency,

$$\begin{aligned} f_{obs} &= \frac{c - v_{obs}}{\lambda_{med}} = \frac{c - v_{obs}}{\frac{c}{f_{fnt}}\left(1 - \frac{v_{fnt}}{c}\right)} = f_{fnt}\frac{1 - \frac{v_{obs}}{c}}{1 - \frac{v_{fnt}}{c}} \\ &\simeq f_{fnt}\left(1 - \frac{v_{obs}}{c}\right)\left(1 + \frac{v_{fnt}}{c}\right) = f_{fnt}\left(1 - \frac{v_{obs} - v_{fnt}}{c} - \frac{v_{obs}v_{fnt}}{c}\right) \\ &\simeq f_{fnt}\left(1 - \frac{v_{obs} - v_{fnt}}{c}\right) \ . \end{aligned}$$