Saturated absorption spectroscopy

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Solution: *The optical density with Doppler broadening is,*

$$\begin{aligned} OD(T,\omega) &= Ln(T)\int_{-\infty}^{\infty} \tfrac{N-2N_e}{N}\sigma(v)\rho(v)dv \\ &= L\frac{P}{k_BT}\sqrt{\frac{m}{2\pi k_BT}}\frac{6\pi}{k^2}\int_{-\infty}^{\infty}\left(1-\frac{2\Omega^2}{4(\Delta+kv)^2+2\Omega^2+\Gamma^2}\right)\times \\ &\qquad \times\frac{\Gamma^2}{4(\Delta-kv)^2+\Gamma^2}e^{-mv^2/2k_BT}dv\ , \end{aligned}$$

with $\Delta\equiv\omega-\omega_0$. *The widths of the three speed distributions are, respectively,*

$$\begin{aligned} k\Delta v &= \sqrt{\tfrac{1}{2}\Omega^2+\tfrac{1}{4}\Gamma^2}\approx(2\pi)\ 68\ MHz \qquad \textit{to the saturation beam} \\ k\bar{v} &= k\sqrt{\tfrac{k_BT}{m}}\approx(2\pi)\ 217\ MHz \qquad \textit{for Doppler enlargement} \\ k\Delta v &= \tfrac{1}{2}\Gamma\approx(2\pi)\ 3\ MHz \qquad \textit{to the test beam}\ . \end{aligned}$$

where $\bar{v}=\sqrt{k_BT/m}$ *is the mean atomic velocity (or the rms width) of the Maxwell distribution. Since the test beam width is much smaller, we can substitute a function* δ*,*

$$\frac{\Gamma^2}{4(\Delta-kv)^2+\Gamma^2}\longrightarrow\frac{\pi\Gamma}{2}\delta(\Delta-kv)\ ,$$

giving

$$\begin{aligned} OD(T,\omega) &\simeq L\frac{P}{k_BT}\sqrt{\frac{m}{2\pi k_BT}}\frac{6\pi}{k^3}\int_{-\infty}^{\infty}\left(1-\frac{2\Omega^2}{4(\Delta+kv)^2+2\Omega^2+\Gamma^2}\right)\times \\ &\qquad \times\frac{\pi\Gamma}{2}\delta(\Delta-kv)e^{-mv^2/2k_BT}d(kv) \\ &= L\frac{P}{k_BT}\sqrt{\frac{m}{2\pi k_BT}}\frac{6\pi}{k^3}\frac{\pi\Gamma}{2}\left(1-\frac{2\Omega^2}{16\Delta^2+2\Omega^2+\Gamma^2}\right)e^{-m(\Delta/k)^2/2k_BT}\ . \end{aligned}$$

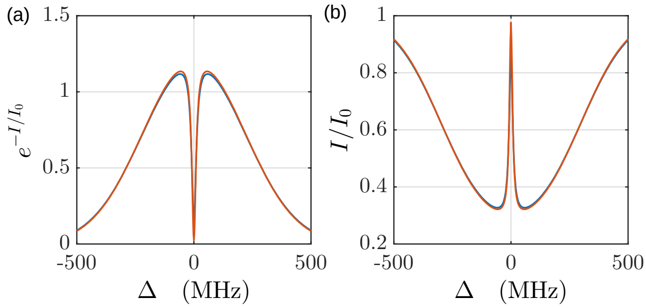

Figure 2.13: (code for download) (a) Optical density and (b) absorption. (Blue) Integral formula and (green) high temperature approach and high saturation.

Consider a tube through which passes a collimated beam of atoms, all having the same initial velocity $v = v_0$. In the opposite direction to the atomic motion travels a collimated and monochromatic light beam with frequency $\omega = kc$. The absorption rate for photons by an atom has a Lorentzian profile, which can be written as:

$$W(v) = \frac{W_0}{2\pi} \frac{\Gamma^2}{(\omega - \omega_0 + kv)^2 + (\Gamma/2)^2} ,$$

where Γ is the natural width of the spectral line at ω_0, and W_0 is a constant. The frequency of the light is tuned in order to compensate for the Doppler effect at the beginning of the tube, $\delta = \omega - \omega_0 = -kv_0$ (the light is tuned to the red of the resonance). As the atoms are decelerated, they cease to be resonant with the light beam and fail to absorb photons. This can be avoided by employing the so-called Zeeman-slowing technique, which compensates for the effect using the Zeeman-shift induced by magnetic fields. In this exercise, we will study what happens if this technique is not used.

a. For an atom with velocity v, write an expression for the mean travel distance

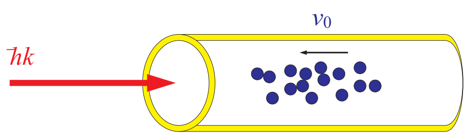

Figure 2.14: Zeeman slower scheme.

$\Delta s(v)$ before it absorbs a photon as a function of the parameters Γ, v_0, k, and W_0. (The mean time it takes to absorb a photon is $W(v)^{-1}$).

b. The velocity of the atom as a function of the number of absorbed photons is $v_n = v_0 - n\frac{\hbar k}{m}$, the second term being the recoil due to the absorption of a single photon. The average total distance traveled by an atom after absorbing N photons is estimated by:

$$S = \sum_{n=0}^{N} \Delta s(v_n) \simeq \int_0^N \Delta s(v_n) dn .$$