## Saturation broadening and Autler-Townes splitting

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Solution: a. The eigenvalues of the effective Hamiltonian (1.38) excited in resonance,

$$E = -\frac{i}{4}\Gamma \pm \frac{i}{4}\sqrt{\Gamma^2 - 4\Omega^2} \; ,$$

describe possible effects of line broadening and/or shift due to the coupling. Two cases are interesting: In the case  $\Gamma > 4\Omega$  we get,

$$E = \mathfrak{Re} \ E = 0$$
 ,  $\Gamma_{eff} = -2\mathfrak{Im} \ E = \frac{1}{2}\Gamma \mp \frac{1}{2}\sqrt{\Gamma^2 - 4\Omega^2}$ .

That is, the resonance is not shifted or split, but undergoes a line broadening, as already shown in (2.76). In the case  $\Gamma < 40$ 

In the case  $\Gamma < 4\Omega$ ,

$$E = \mathfrak{Re} \ E = \pm \frac{1}{2} \sqrt{\Omega^2 - \frac{1}{4} \Gamma^2} \quad , \qquad \Gamma_{eff} = -2 \mathfrak{Im} \ E = \frac{1}{2} \Gamma \ .$$

we observe an splitting of the line called Auther-Townes splitting. When saturation is strong, the two new lines are separated by  $\Omega$ , each having the natural width  $\Gamma$ . Figs. 2.15(a,b) show the bifurcation of the spectrum at the point  $\Omega_{12} = \frac{1}{2}\Gamma$ .

b. The Liouville matrix can be found in the numerical MATLAB code in given in the file 'LM\_Bloch\_AutlerTownes.m'.

c. Fig. 2.15(c) shows the results of the simulations. The laser  $\Omega_{23}$  probes the population  $\rho_{22}$  by excitation to a higher level, that is, the fluorescence emitted by the population  $\rho_{33}$  is representative for the population  $\rho_{22}$ .



Figure 2.15: (code for download) (a) Autler-Townes splitting and (b) linewidths as a function of the Rabi frequency. (c) Population of the excited state in a three-level system in cascade configuration, as shown in Fig. 2.3(c) in a function of the Rabi frequencies  $\Omega_{23}$  and  $\Omega_{12}$ . The parameters are,  $\Gamma_{23} = 0.5\Gamma_{12}$ ,  $\Gamma_{13} = 0.01\Gamma_{12}$ ,  $\Omega_{23} = 0.1\Gamma_{12}$ .