## Saturation broadening and Autler-Townes splitting

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Solution: $a$. The eigenvalues of the effective Hamiltonian (1.38) excited in resonance,

$$
E=-\frac{\imath}{4} \Gamma \pm \frac{\imath}{4} \sqrt{\Gamma^{2}-4 \Omega^{2}},
$$

describe possible effects of line broadening and/or shift due to the coupling. Two cases are interesting: In the case $\Gamma>4 \Omega$ we get,

$$
E=\mathfrak{R e} E=0 \quad, \quad \Gamma_{e f f}=-2 \mathfrak{I m} E=\frac{1}{2} \Gamma \mp \frac{1}{2} \sqrt{\Gamma^{2}-4 \Omega^{2}} .
$$

That is, the resonance is not shifted or split, but undergoes a line broadening, as already shown in (2.76).
In the case $\Gamma<4 \Omega$,

$$
E=\mathfrak{R e} E= \pm \frac{1}{2} \sqrt{\Omega^{2}-\frac{1}{4} \Gamma^{2}} \quad, \quad \Gamma_{e f f}=-2 \mathfrak{I m} E=\frac{1}{2} \Gamma .
$$

we observe an splitting of the line called Autler-Townes splitting. When saturation is strong, the two new lines are separated by $\Omega$, each having the natural width $\Gamma$. Figs. 2.15 $(a, b)$ show the bifurcation of the spectrum at the point $\Omega_{12}=\frac{1}{2} \Gamma$.
b. The Liouville matrix can be found in the numerical MATLAB code in given in the file 'LM_Bloch_AutlerTownes.m'.
c. Fig. 2.15(c) shows the results of the simulations. The laser $\Omega_{23}$ probes the population $\rho_{22}$ by excitation to a higher level, that is, the fluorescence emitted by the population $\rho_{33}$ is representative for the population $\rho_{22}$.


Figure 2.15: (code for download) (a) Autler-Townes splitting and (b) linewidths as a function of the Rabi frequency. (c) Population of the excited state in a three-level system in cascade configuration, as shown in Fig. 2.3(c) in a function of the Rabi frequencies $\Omega_{23}$ and $\Omega_{12}$. The parameters are, $\Gamma_{23}=0.5 \Gamma_{12}, \Gamma_{13}=0.01 \Gamma_{12}, \Omega_{23}=0.1 \Gamma_{12}$.

