Derivation of Bloch equations

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Solution: *We differentiate the density operator,*

$$\frac{d\rho_{mn}}{dt} = a_m \frac{da_n^*}{dt} + \frac{da_m}{dt} a_n^* \ .$$

Substituting the derivatives of the population amplitudes by (1.18) and the transition elements by (1.19),

$$\frac{da_1}{dt} = -\imath\Omega \cos\omega t e^{-i\omega_0 t} a_2 \qquad \text{and} \qquad \frac{da_2}{dt} = -\imath\Omega^* \cos\omega t e^{i\omega_0 t} a_1$$

for the time derivatives of (2.50) and letting $\Omega = \Omega^*$,

$$\begin{aligned}\frac{d\rho_{22}}{dt} &= \imath\Omega \cos\omega t e^{-\imath\omega_0 t} a_2 a_1^* - \imath\Omega^* \cos\omega t e^{\imath\omega_0 t} a_1 a_2^* = \imath\Omega_0 \frac{e^{\imath\omega t} + e^{-\imath\omega t}}{2} \left(a_2 a_1^* e^{-\imath\omega_0 t} - a_1 a_2^* e^{\imath\omega_0 t}\right) \\ &\simeq \frac{\imath\Omega}{2} \left(a_2 a_1^* e^{\imath\Delta t} - a_1 a_2^* e^{-\imath\Delta t}\right) = \frac{\imath\Omega}{2} \left(\rho_{21} e^{\imath\Delta t} - \rho_{12} e^{-\imath\Delta t}\right) = -\frac{d\rho_{11}}{dt} \ .\end{aligned}$$

and

$$\begin{aligned}\frac{d\rho_{12}}{dt} &= \imath\Omega \cos\omega t e^{-\imath\omega_0 t} a_1 a_1^* - \imath\Omega \cos\omega t e^{-\imath\omega_0 t} a_2 a_2^* = \imath\Omega \frac{e^{\imath\omega t} + e^{-\imath\omega t}}{2} e^{-\imath\omega_0 t} (|a_1|^2 - |a_2|^2) \\ &\simeq \frac{\imath\Omega}{2} |a_1|^2 e^{\imath\Delta t} = \frac{\imath\Omega}{2} (\rho_{11} - \rho_{22}) e^{\imath\Delta t} = \frac{d\rho_{21}^*}{dt} \ .\end{aligned}$$

In this form the Bloch equations are in the interaction representation.