Photon echo

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Solution: *a. Choosing $\Omega_{12} = 1$ the duration of the $\pi/2$-pulse must be $T = \pi/2\Omega_{12}$. The Hamiltonian and the solution of the Schrödinger equation can be found in the numerical MATLAB code given in the file 'LM_Bloch_PhotonEcho.m'. Fig. 2.11(a) shows, that the final inversion $2\rho_{22}(2T+\tau)-1$ depends on the detuning Δ_{12}, and the free precession time τ between the pulses*
b. The code 'LM_Bloch_PhotonEcho.m' shows the simulations.

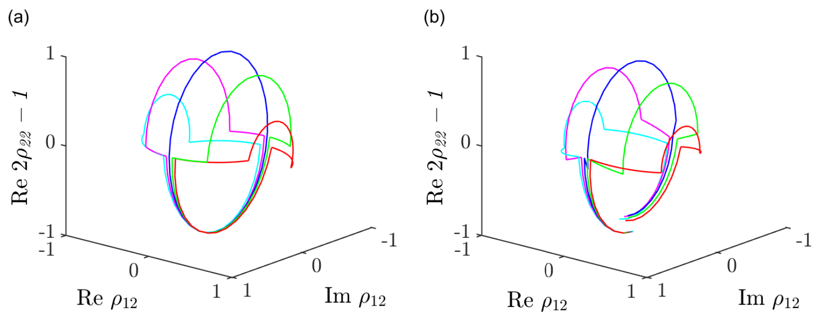

Figure 2.11: (code for download) (a) Time evolution of the Bloch vector for different detunings Δ_{12} calculated from the Schrödinger equation. (b) Same evolution now calculated from the Bloch equations with $\Gamma_{12} = 0.1\Omega_{12}$ the spontaneous emission rate.

Alternatively we can try an analytical approach: *We use the general solutions of the Bloch equations:*

$$\begin{aligned}
\rho_{22}^{(1)} &= \rho_{22}(0) + \tfrac{1}{2G^2}\left[|\Omega|^2(1-2\rho_{22}(0)) - \Delta(\Omega\rho_{12}^*(0) + \Omega^*\rho_{12}(0))\right] \\
\rho_{22}^{(2)} &= \tfrac{1}{4G^2}\left[-|\Omega|^2(1-2\rho_{22}(0)) + (\Delta+G)\Omega\rho_{12}^*(0) + (\Delta-G)\Omega^*\rho_{12}(0)\right] \\
\rho_{22}^{(3)} &= \tfrac{1}{4G^2}\left[-|\Omega|^2(1-2\rho_{22}(0)) + (\Delta-G)\Omega\rho_{12}^*(0) + (\Delta+G)\Omega^*\rho_{12}(0)\right] \\
\rho_{12}^{(1)} &= \tfrac{1}{2G^2}\left[\Delta\Omega(1-2\rho_{22}(0)) + \Omega\left(\Omega\rho_{12}^*(0) + \Omega^*\rho_{12}(0)\right)\right] \\
\rho_{12}^{(2)} &= \tfrac{\Delta-G}{4G^2}\left[-\Omega(1-2\rho_{22}(0)) + (\Delta+G)\frac{\Omega}{\Omega^*}\rho_{12}^*(0) + (\Delta-G)\rho_{12}(0)\right] \\
\rho_{12}^{(3)} &= \tfrac{\Delta+G}{4G^2}\left[-\Omega(1-2\rho_{22}(0)) + (\Delta-G)\frac{\Omega}{\Omega^*}\rho_{12}^*(0) + (\Delta+G)\rho_{12}(0)\right] ,
\end{aligned}$$

and consider the special cases $\Delta = 0$,

$$\begin{array}{lll}
\rho_{22}^{(1)} = \frac{1}{2} & \rho_{22}^{(2)} = -\frac{1}{4} + \frac{1}{2}\rho_{22}(0) + \frac{i}{2}\mathfrak{Im}\ \rho_{12}(0) & \rho_{22}^{(3)} = -\frac{1}{4} + \frac{1}{2}\rho_{22}(0) - \frac{i}{2}\mathfrak{Im}\ \rho_{12}(0) \\
\rho_{12}^{(1)} = \mathfrak{Re}\ \rho_{12}(0) & \rho_{12}^{(2)} = \frac{1}{4} - \frac{1}{2}\rho_{22}(0) + \frac{i}{2}\mathfrak{Im}\ \rho_{12}(0) & \rho_{12}^{(3)} = -\frac{1}{4} + \frac{1}{2}\rho_{22}(0) - \frac{i}{2}\mathfrak{Im}\ \rho_{12}(0)
\end{array}$$

with the solution

$$\begin{aligned}\rho_{22}(t) &= \rho_{22}^{(1)} + \rho_{22}^{(2)} e^{\imath\Omega t} + \rho_{22}^{(3)} e^{-\imath\Omega t}\\ &= \tfrac{1}{2} + \left[-\tfrac{1}{4} + \tfrac{1}{2}\rho_{22}(0) - \tfrac{\imath}{2}\mathfrak{Im}\ \rho_{12}(0)\right] e^{\imath\Omega t} + \left[-\tfrac{1}{4} + \tfrac{1}{2}\rho_{22}(0) + \tfrac{\imath}{2}\mathfrak{Im}\ \rho_{12}(0)\right] e^{-i\Omega t}\\ \rho_{12}(t) &= \rho_{12}^{(1)} + \rho_{12}^{(2)} e^{\imath\Omega t} + \rho_{12}^{(3)} e^{-\imath\Omega t}\\ &= \mathfrak{Re}\ \rho_{12}(0) + \left[\tfrac{1}{4} - \tfrac{1}{2}\rho_{22}(0) + \tfrac{\imath}{2}\mathfrak{Im}\ \rho_{12}(0)\right] e^{\imath\Omega t} + \left[-\tfrac{1}{4} + \tfrac{1}{2}\rho_{22}(0) + \tfrac{\imath}{2}\mathfrak{Im}\ \rho_{12}(0)\right] e^{-\imath\Omega t}\end{aligned}$$

and the case $\Omega = 0$

$$\begin{array}{lll}\rho_{22}^{(1)} = \rho_{22}(0) & \rho_{22}^{(2)} = 0 & \rho_{22}^{(3)} = 0\\ \rho_{12}^{(1)} = 0 & \rho_{12}^{(2)} = 0 & \rho_{12}^{(3)} = \rho_{12}(0)\end{array}$$

with the solution

$$\begin{aligned}\rho_{22}(t) &= \rho_{22}^{(1)} + \rho_{22}^{(1)} e^{\imath\Delta t} + \rho_{22}^{(1)} e^{-\imath\Delta t} = \rho_{22}(0)\\ \rho_{12}(t) &= \rho_{12}^{(1)} + \rho_{12}^{(1)} e^{\imath\Delta t} + \rho_{12}^{(1)} e^{-\imath\Delta t} = \rho_{12}(0) e^{-\imath\Delta t}\ .\end{aligned}$$

We begin with the initial conditions

$$\begin{aligned}\rho_{22}(0) &= 0\\ \rho_{12}(0) &= 0\ .\end{aligned}$$

We apply a first resonant $\pi/2$*-pulse* ($\Omega t = \pi/2, \Delta = 0$),

$$\begin{aligned}\rho_{22}(\tfrac{\pi}{2\Omega}) &= \tfrac{1}{2} - \tfrac{1}{2}\cos\Omega t = \tfrac{1}{2}\\ \rho_{12}(\tfrac{\pi}{2\Omega}) &= \tfrac{\imath}{2}\sin\Omega t = \tfrac{\imath}{2}\ .\end{aligned}$$

Now, we turn off the laser and detune the transition ($\Omega = 0, \Delta_T$)*. After an arbitrary time* T *the state becomes,*

$$\begin{aligned}\rho_{22}(\tfrac{\pi}{2\Omega} + T) &= \frac{1}{2}\\ \rho_{12}(\tfrac{\pi}{2\Omega} + T) &= \frac{\imath}{2} e^{-\imath\Delta_T T} = \tfrac{1}{2}\sin\Delta_T T + \tfrac{\imath}{2}\cos\Delta_T T\ .\end{aligned}$$

Now we apply a resonant π*-pulse* ($\Omega t = \pi, \Delta = 0$),

$$\begin{aligned}\rho_{22}(\tfrac{\pi}{2\Omega} + T + \pi) &= \tfrac{1}{2} - \tfrac{1}{2}\cos\Delta_T T \sin\Omega t = \tfrac{1}{2}\\ \rho_{12}(\tfrac{\pi}{2\Omega} + T + \pi) &= \tfrac{1}{2}\sin\Delta_T T + \tfrac{\imath}{2}\cos\Delta_T T\cos\Omega t = \tfrac{1}{2}\sin\Delta_T T - \tfrac{i}{2}\cos\Delta_T T = -\frac{\imath}{2} e^{\imath\Delta_T T}\ .\end{aligned}$$

We let the system evolve once more for a while T *without laser* ($\Omega = 0, \Delta_T$),

$$\begin{aligned}\rho_{22}(\tfrac{\pi}{2\Omega} + T + \pi + T) &= \frac{1}{2}\\ \rho_{12}(\tfrac{\pi}{2\Omega} + T + \pi + T) &= -\frac{i}{2} e^{i\Delta_T T} e^{-i\Delta_T t} = -\frac{\imath}{2}\ .\end{aligned}$$

Finally, we apply the second $\pi/2$*-pulse* ($\Omega t = \pi/2, \Delta = 0$)

$$\begin{aligned}\rho_{22}(\tfrac{\pi}{2\Omega} + T + \tfrac{\pi}{\Omega} + T + \tfrac{\pi}{2\Omega}) &= \tfrac{1}{2} + \tfrac{1}{2}\sin\Omega t = 1\\ \rho_{12}(\tfrac{\pi}{2\Omega} + T + \tfrac{\pi}{\Omega} + T + \tfrac{\pi}{2\Omega}) &= \dots\ .\end{aligned}$$

Therefore, for each value of them detuning Δ_T, which may in fact be different for each member of an atomic sample, the final value of the population of the excited state is always 1.

Photon echo is an interference effect in the radiation emitted by many atoms. We assume that many two-level atoms interact with the same radiation mode. To observe photon echoes, we apply a first $\pi/2$-pulse which brings every atom into a coherent superposition of the two energy states. However, because of the Doppler shift, the excitation of the individual atoms may be different, so that the individual Bloch vectors begin to precess at different paces. If after a certain time a π-pulse is applied, the state populations, and therefore the direction of the precession are reversed, so that after a certain time, the Bloch vectors are refocused.

The synchronization of the phases of the individual dipoles generates a cooperative spontaneous emission of the ensemble called photon echo. The signature of a photon echo is twofold: First, the appearance of a fluorescence pulse a delay of τ after the end of the applied π-pulse, and second, a fluorescence rate depending on the square of the excited state population. This unusual behavior emerges from a mutual coupling of the individual dipoles and results in a rapid depopulation of the excited state with a much shorter fluorescent lifetime than for individual atoms. This synchronization of the phases of the individual dipoles is called superradiance. It is important to keep in mind, that the photon echo does not allow to recover the coherence of an irreversible process. It only works for inhomogeneous broadening due to a well-defined distribution of the kinetic energy among the atoms, provided that the temporal evolutions of the individual atoms have not undergone random phase interruptions.