General form of the master equation

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Solution: *Inserting the Hamiltonian*

$$H = \begin{pmatrix} 0 & \frac{1}{2}\hbar\Omega \\ \frac{1}{2}\hbar\Omega & \hbar\Delta \end{pmatrix}$$

into the master equation, we obtain,

$$-\frac{\imath}{\hbar}[H,\rho] = -\imath \begin{pmatrix} \frac{1}{2}\Omega\rho_{21} - \frac{1}{2}\Omega\rho_{12} & \frac{1}{2}\Omega\rho_{22} - \frac{1}{2}\Omega\rho_{11} - \rho_{12}\Delta \\ \frac{1}{2}\Omega\rho_{11} + \Delta\rho_{21} - \frac{1}{2}\Omega\rho_{22} & \frac{1}{2}\Omega\rho_{12} - \frac{1}{2}\Omega\rho_{21} \end{pmatrix}$$

and,

$$\frac{\Gamma}{2}\left(2\sigma\rho\sigma^+ - \sigma^+\sigma\rho - \rho\sigma^+\sigma\right) = \tfrac{1}{2}\Gamma \begin{pmatrix} 2\rho_{22} & -\rho_{12} \\ -\rho_{21} & -2\rho_{22} \end{pmatrix} \ .$$

Thereby,

$$\frac{d\rho}{dt} = \begin{pmatrix} -\frac{\imath}{2}\Omega(\rho_{21} - \rho_{12}) + \Gamma\rho_{22} & -\imath\left(\frac{1}{2}\Omega\rho_{22} - \frac{1}{2}\Omega\rho_{11} - \rho_{12}\Delta\right) - \frac{1}{2}\Gamma\rho_{12} \\ -\imath\left(\frac{1}{2}\Omega\rho_{11} + \Delta\rho_{21} - \frac{1}{2}\Omega\rho_{22}\right) - \frac{1}{2}\Gamma\rho_{21} & -\frac{\imath}{2}\Omega(\rho_{12} - \rho_{21}) - \Gamma\rho_{22} \end{pmatrix} \ ,$$

or,

$$\frac{d}{dt}\begin{pmatrix} \rho_{11} \\ \rho_{22} \\ \rho_{12} \\ \rho_{21} \end{pmatrix} = \begin{pmatrix} 0 & \Gamma & \frac{\imath}{2}\Omega & -\frac{\imath}{2}\Omega \\ 0 & -\Gamma & -\frac{\imath}{2}\Omega & \frac{\imath}{2}\Omega \\ \frac{\imath}{2}\Omega & -\frac{\imath}{2}\Omega & -\frac{1}{2}\Gamma + \imath\Delta & 0 \\ -\frac{\imath}{2}\Omega & \frac{\imath}{2}\Omega & 0 & -\frac{1}{2}\Gamma - \imath\Delta \end{pmatrix} \begin{pmatrix} \rho_{11} \\ \rho_{22} \\ \rho_{12} \\ \rho_{21} \end{pmatrix} \ .$$