

Mixture of states

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Solution: a. The state is,

$$\hat{\rho} = \frac{1}{2}|1\rangle\langle 1| + \frac{1}{2}|2\rangle\langle 2| .$$

Obviously, $\hat{\rho}^2 \neq \hat{\rho}$, mas $\text{Tr } \hat{\rho} = 1$.

b. The probability is,

$$\langle \hat{P}_1 \rangle = \text{Tr } \hat{\rho} \hat{P}_1 = \sum_k \langle k | \hat{\rho} \hat{P}_1 | k \rangle = \sum_k \langle k | (\frac{1}{2}|1\rangle\langle 1| + \frac{1}{2}|2\rangle\langle 2|) |1\rangle\langle 1| k \rangle = \frac{1}{2} .$$