Thermal mixture

Philippe W. Courteille, 27/12/2021

Solution: *a. The probability of measuring exactly the velocity v' for an atom is,*

$$\langle v'|\hat{\rho}|v'\rangle = \langle v'|\int dv\sqrt{\frac{m}{2\pi k_BT}}e^{-mv^2/2k_BT}|v\rangle\langle v|v'\rangle$$
$$= \int dv\sqrt{\frac{m}{2\pi k_BT}}e^{-mv^2/2k_BT}\delta(v-v') = \sqrt{\frac{m}{2\pi k_BT}}e^{-mv'^2/2k_BT}\ .$$

b. The expectation value is,

$$\langle\hat{v}^2\rangle = Tr\ \rho v^2 = \int du\langle u|\rho\hat{v}^2|u\rangle = \int duu^2\langle u|\rho|u\rangle = \sqrt{\frac{m}{2\pi k_BT}}\int duu^2\langle u|\int dve^{-mv^2/2k_BT}|v\rangle\langle v|u\rangle$$
$$= \sqrt{\frac{m}{2\pi k_BT}}\int duu^2\langle u|e^{-mu^2/2k_BT}|u\rangle = \sqrt{\frac{m}{2\pi k_BT}}\int u^2e^{-mu^2/2k_BT}du$$
$$= \sqrt{\frac{m}{2\pi k_BT}}\left(\frac{2k_BT}{m}\right)^{3/2}\int_{-\infty}^{\infty}z^2e^{-z^2}dz = \sqrt{\frac{1}{\pi}}\frac{2k_BT}{m}\frac{\sqrt{\pi}}{2} = \frac{k_BT}{m}\ .$$