Dispersive interaction between an atom and light

Philippe W. Courteille, 24/02/2023

Solution: *a. The Hamiltonian for the system is,*

$$\hat{H}_{12} = \begin{pmatrix} 0 & \frac{1}{2}\Omega_{12} & 0 \\ \frac{1}{2}\Omega_{12} & 0 & \frac{1}{2}\Omega_{23} \\ 0 & \frac{1}{2}\Omega_{23} & \Delta_{23} \end{pmatrix} .$$

b. The Hamiltonian of the subsystem of the two upper levels is,

$$\hat{H}_{23} = \begin{pmatrix} 0 & \frac{1}{2}\Omega_{23} \\ \frac{1}{2}\Omega_{23} & \Delta_{23} \end{pmatrix} .$$

When excited very far-off resonance, we can expand its eigenvalues for small Ω_{23}*,*

$$E_{1,2} = \tfrac{1}{2}(\Delta_{23} \pm G_{23}) = \frac{1}{2}\Delta_{23} \pm \frac{1}{2}\Delta_{23}\left(1 + \frac{\Omega_{23}^2}{2\Delta_{23}}\right) .$$

By substituting the matrix of corrected eigenvalues in the complete Hamiltonian, we obtain,

$$\hat{H} \simeq \begin{pmatrix} 0 & \frac{1}{2}\Omega_{12} & 0 \\ \frac{1}{2}\Omega_{12} & -\frac{\Omega_{23}^2}{4\Delta_{23}} & 0 \\ 0 & 0 & \Delta_{23} + \frac{\Omega_{23}^2}{4\Delta_{23}} \end{pmatrix} .$$

c. We simulate the evolution of the state to be,

$$|\psi(t)\rangle = e^{\imath\hat{H}^{(3)}\Delta t_3/\hbar} e^{\imath\hat{H}^{(2)}\Delta t_2/\hbar} e^{\imath\hat{H}^{(1)}\Delta t_1/\hbar} |\psi_(0)\rangle .$$

The numerical MATLAB code is given in the file 'LM_Bloch_Dispersive'.
d. We simulate the evolution of the Bloch vector to be,

$$\vec{\rho}(t) = e^{\mathcal{L}_3\Delta t_3} e^{\mathcal{L}_2\Delta t_2} e^{\mathcal{L}_1\Delta t_1} \vec{\rho}(0) .$$

The numerical MATLAB code is given in the file 'LM_Bloch_Dispersive'. Fig. 2.22 shows the result of the simulations. We find that Δ_{23} *must be chosen very large (*> 300*) to obtain a really dispersive dynamics.*

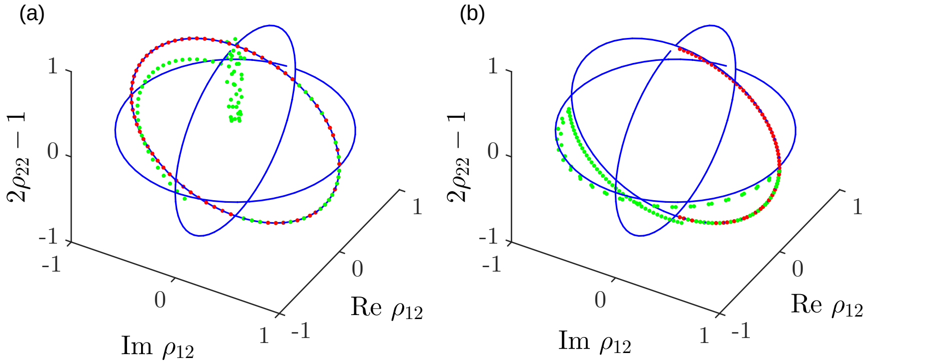

Figure 2.22: (code for download) (red) Evolution of Bloch vector for a sequence of resonant Ramsey pulses. (blue) Evolution for the occurrence of a dispersive pulse on a transition coupling an upper level.