Light-shift

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Solution: *a. The eigenvalues of the effective Hamiltonian (1.40),*

$$E = \tfrac{1}{2}\Lambda \pm \tfrac{1}{2}G_\Gamma \qquad \textit{with} \qquad \Lambda \equiv \Delta - \tfrac{i}{2}\Gamma \qquad \textit{and} \qquad G_\Gamma \equiv \sqrt{\Omega^2 + \Lambda^2}$$

describe the dynamic Stark shift. Expanding for weak coupling, $\Gamma \gg \Omega$*,*

$$E = \tfrac{\Lambda}{2} \pm \tfrac{1}{2}\sqrt{\Lambda^2 + \Omega^2} \overset{\Omega \ll \Gamma}{\longrightarrow} \tfrac{\Lambda}{2} \pm \tfrac{\Lambda}{2}\left(1 + \tfrac{\Omega^2}{2\Lambda^2}\right) = \tfrac{\Lambda}{2} \pm \tfrac{\Lambda}{2}(1+s) \ ,$$

with the definition (2.76) of the saturation parameter. Expanding for strong coupling, $\Gamma \ll \Omega$*,*

$$E = Re\ \left(\tfrac{1}{2}\Lambda \pm \tfrac{1}{2}G_\Gamma\right) = \tfrac{1}{2}\Delta \pm \tfrac{1}{2}\sqrt{\sqrt{\tfrac{1}{4}G^4 + \tfrac{1}{8}\Gamma^2\left(G^2 - 2\Omega^2 + \tfrac{1}{8}\Gamma^2\right)} + \tfrac{1}{2}G^2 - \tfrac{1}{8}\Gamma^2}$$
$$\overset{\Omega \gg \Gamma}{\longrightarrow} \tfrac{1}{2}\Delta \pm \tfrac{1}{2}G \mp \tfrac{\Gamma^2\Omega^2}{16G^3} \ ,$$

where G *is the common generalized Rabi frequency. For big detunings we can approximate,*

$$E \overset{\Gamma \to 0}{\simeq} \tfrac{1}{2}\Delta \pm \tfrac{1}{2}G = \tfrac{1}{2}\Delta \pm \tfrac{\Delta}{2} \mp \tfrac{\Omega^2}{4\Delta} + \ldots \ .$$

b. The Liouville matrix can be found in the numerical MATLAB code given in the file 'LM_Bloch_LightShift.m'. Fig. 2.11 shows the results of the simulations. The dynamic Stark splitting $E_1 - E_2$ *is a consequence of the breaking of the degeneracy of the states dressed* $|1,N\rangle$ *and* $|2,N-1\rangle$ *for strong fields* $\Gamma \ll \Omega$*. The light shift is the expectation value of the perturbation of the atom by the light field. While the positions of the resonances follow from Re* (E)*, the linewidth follows from Im* (E)*. In the weak coupling regime, we observe a dispersive dependence of the energy shift. In the strong coupling regime we observe an avoided crossing.*

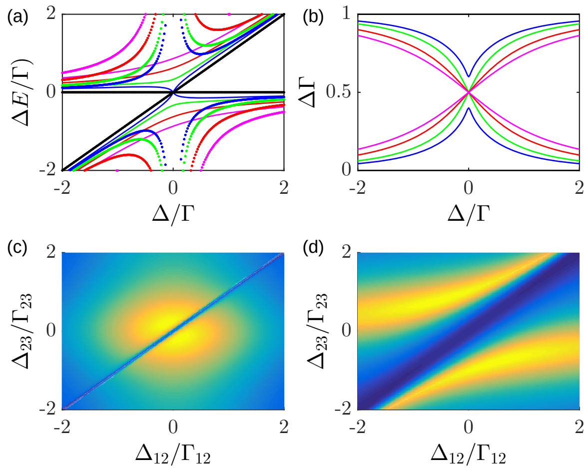

Figure 2.11: (code for download) (a) Light shift in a two level system varying Ω_{12} between $0.5\Gamma_{12}$ (internal blue curve) and $1\Gamma_{12}$ (external magenta curve). (b) Linewidth for the same parameters as in (a). (c,d) Population of the excited state in a three-level system in Λ-configuration, as shown in Fig. 2.3(a) sweeping the lasers. These figures are discussed later. For (c) $\Omega_{12}/\Gamma_{12} = 0.2$ and for (d) $\Omega_{12}/\Gamma_{12} = 2$.