

Normalization of the Bloch vector

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Solution: *The Bloch vector is,*

$$\vec{\sigma} = (2\Re \rho_{12} \quad 2\Im \rho_{12} \quad \rho_{22} - \rho_{11}) = (\rho_{12} + \rho_{21} \quad -i(\rho_{12} - \rho_{21}) \quad \rho_{22} - \rho_{11}) .$$

The length of the vector is,

$$\sqrt{\vec{\sigma}^t \vec{\sigma}} = \sqrt{4\rho_{12}\rho_{21} + \rho_{22}^2 - 2\rho_{22}\rho_{11} + \rho_{11}^2} = \sqrt{\rho_{22}^2 + 2\rho_{22}\rho_{11} + \rho_{11}^2} = 1 ,$$

using $\det \rho = \rho_{22}\rho_{11} - \rho_{12}\rho_{21} = 0$ *and* $\text{Tr} \rho = \rho_{11} + \rho_{22} = 1$.