Sequence of Ramsey pulses

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Solution: *To describe the dynamics of a two-level atom during a Ramsey cycle let us consider the general solutions (2.54) for two specific cases. For $\Omega = 0$, we get*

$$\rho_{22}^{(1)} = \rho_{22}(0) \quad \textit{and} \quad \rho_{12}^{(3)} = \tilde{\rho}_{12}(0) \quad \textit{and} \quad \rho_{22}^{(2)} = 0 = \rho_{22}^{(3)} = \rho_{12}^{(1)} = \rho_{12}^{(2)} \ .$$

Hence,

$$\begin{aligned}\rho_{22}(t) &= \rho_{22}^{(1)} + \rho_{22}^{(2)} e^{\imath Gt} + \rho_{22}^{(3)} e^{-\imath Gt} = \rho_{22}(0)\\ \tilde{\rho}_{12}(t) &= \rho_{12}^{(1)} + \rho_{12}^{(2)} e^{\imath Gt} + \rho_{12}^{(3)} e^{-\imath Gt} = \tilde{\rho}_{12}(0) e^{-\imath\Delta t} \ .\end{aligned}$$

For $\Delta = 0$, we get

$$\begin{aligned}\rho_{22}^{(1)} &= \tfrac{1}{2}\\ \rho_{22}^{(2)} &= \tfrac{1}{4}\left[-(1-2\rho_{22}(0)) + \tilde{\rho}_{12}^*(0) - \tilde{\rho}_{12}(0)\right]\\ \rho_{22}^{(3)} &= \tfrac{1}{4}\left[-(1-2\rho_{22}(0)) - \tilde{\rho}_{12}^*(0) + \tilde{\rho}_{12}(0)\right]\\ \rho_{12}^{(1)} &= \tfrac{1}{2}\left[\rho_{12}^*(0) + \rho_{12}(0)\right]\\ \rho_{12}^{(2)} &= \tfrac{1}{4}\left[(1-2\rho_{22}(0)) - \tilde{\rho}_{12}^*(0) + \tilde{\rho}_{12}(0)\right]\\ \rho_{12}^{(3)} &= \tfrac{1}{4}\left[-(1-2\rho_{22}(0)) - \tilde{\rho}_{12}^*(0) + \tilde{\rho}_{12}(0)\right] \ .\end{aligned}$$

Hence,

$$\begin{aligned}\rho_{22}(t) &= \rho_{22}^{(1)} + \rho_{22}^{(2)} e^{\imath\Omega t} + \rho_{22}^{(3)} e^{-\imath\Omega t} = \tfrac{1}{2} - \tfrac{1}{2}\left[1-2\rho_{22}(0)\right]\cos\Omega t + \mathfrak{Im}\ \rho_{12}(0)\sin\Omega t\\ \tilde{\rho}_{12}(t) &= \rho_{12}^{(1)} + \rho_{12}^{(2)} e^{\imath\Omega t} + \rho_{12}^{(3)} e^{-\imath\Omega t} = \mathfrak{Re}\ \tilde{\rho}_{12}(0) + \tfrac{\imath}{2}\left[1-2\rho_{22}(0)\right]\sin\Omega t + \imath\mathfrak{Im}\ \tilde{\rho}_{12}(0)\cos\Omega t \ .\end{aligned}$$

At the beginning the atom is in $\rho_{22}(0) = 0 = \tilde{\rho}_{12}(0)$. For this case, the equations simplify to,

$$\rho_{22}(t) = \tfrac{1}{2} - \tfrac{1}{2}\cos\Omega t \quad \textit{and} \quad \tilde{\rho}_{12}(t) = \tfrac{\imath}{2}\sin\Omega t \ .$$

The application of the first $\pi/2$-pulse raises the population of the state to $\rho_{11}(0) = \rho_{22}(0) = \frac{1}{2}$ and the coherence is $\rho_{12}(0) = \frac{\imath}{2}$. We now imagine that the laser source is a little detuned, so little that the population of the coupled state is not affected. But if $\Delta \neq 0$, looking at solutions for $\Omega = 0$, we find that the Bloch vector begins to precess. After a time T, it rotated by a certain angle $\phi \equiv T\Delta$. After this time, we turn on the laser a second time and apply a second $\pi/2$-pulse. We look now at the solutions for $\Delta = 0$, using the new starting conditions, $\rho_{22}(0) = \frac{1}{2}$ and $\rho_{12}(0) = \frac{\imath}{2}e^{-\imath\phi}$,

$$\begin{aligned}\rho_{22}(t) &= \tfrac{1}{2} + \mathfrak{Im}\ \tfrac{\imath e^{-\imath\phi}}{2}\sin\Omega t = \tfrac{1}{2} + \tfrac{1}{2}\cos\phi\\ \tilde{\rho}_{12}(t) &= \mathfrak{Re}\ \tfrac{\imath e^{-\imath\phi}}{2} + \imath\mathfrak{Im}\ \tfrac{\imath e^{-\imath\phi}}{2}\cos\Omega t = \tfrac{1}{2}\sin\phi \ .\end{aligned}$$

That is, we find a phase-dependent oscillation of the excited state population between 0 and 1. Thus, this population, which can be measured experimentally, provides information about the laser detuning.

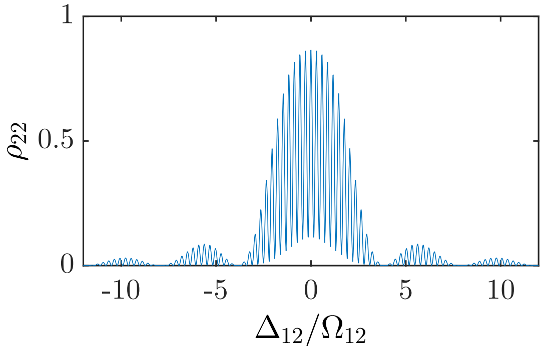

Figure 2.10: (code for download) Ramsey fringes.

Derive the stationary solution of the Bloch equations including spontaneous emission.