Rate equations as a limiting case of Bloch equations

Philippe W. Courteille, 24/02/2023

Solution: *The Einstein rate equations can be derived from the Bloch equations following the Wilcox-Lamb procedure [236, 1]. This consists in applying the condition* $\dot\rho_{12} = 0$ *to the Bloch equation,*

$$\frac{d}{dt}\begin{pmatrix}\rho_{11}\\ \rho_{22}\\ \rho_{12}\\ \rho_{21}\end{pmatrix} = \begin{pmatrix} 0 & \Gamma & \frac{\imath}{2}\Omega & -\frac{\imath}{2}\Omega \\ 0 & -\Gamma & -\frac{\imath}{2}\Omega & \frac{\imath}{2}\Omega \\ \frac{\imath}{2}\Omega & -\frac{\imath}{2}\Omega & \imath\Delta-\gamma & 0 \\ -\frac{\imath}{2}\Omega & \frac{\imath}{2}\Omega & 0 & -\imath\Delta-\gamma \end{pmatrix}\begin{pmatrix}\rho_{11}\\ \rho_{22}\\ \rho_{12}\\ \rho_{21}\end{pmatrix} ,$$

giving,

$$\rho_{12}(\infty) = \frac{-\Omega/2}{\imath\gamma+\Delta}(\rho_{11}-\rho_{22}) .$$

Substituting the coherences in the equations for the populations yields,

$$\frac{d}{dt}\rho_{11} = -\frac{\gamma\Omega^2/2}{\Delta^2+\gamma^2}\rho_{11} + \Gamma\rho_{22} + \frac{\gamma\Omega^2/2}{\Delta^2+\gamma^2}\rho_{22}$$

with the abbreviation,

$$R \equiv \frac{\gamma\Omega^2/2}{\Delta^2+\gamma^2} = \gamma s ,$$

where s is the saturation parameter. We obtain the rate equations,

$$\frac{d}{dt}\begin{pmatrix}\rho_{11}\\ \rho_{22}\end{pmatrix} = \begin{pmatrix} -R & \Gamma+R \\ R & -\Gamma-R \end{pmatrix}\begin{pmatrix}\rho_{11}\\ \rho_{22}\end{pmatrix} .$$