

Trace of an operator

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Solution: a. The basis transformation $|n\rangle = \sum_m |m\rangle \langle m|n\rangle$ applied to the trace gives,

$$\text{Tr } \hat{A} = \sum_n \langle n|\hat{A}|n\rangle = \sum_{n,m,m'} \langle n|m\rangle \langle m|\hat{A}|m'\rangle \langle m'|n\rangle = \sum_{m,m'} \langle m'|m\rangle \langle m|\hat{A}|m'\rangle = \sum_m \langle m|\hat{A}|m\rangle.$$

b. Holds $\text{Tr } \hat{A}\hat{B} = \sum_n \langle n|\hat{A}\hat{B}|n\rangle = \sum_{n,m} \langle n|\hat{A}|m\rangle \langle m|\hat{B}|n\rangle = \text{Tr } \hat{B}\hat{A}.$