

Bloch vector and Bloch equations

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Solution: *The explicit equations of motion for the components of the Bloch vector are,*

$$\frac{d}{dt} \begin{pmatrix} \rho_{12} + \rho_{21} \\ -i(\rho_{12} - \rho_{21}) \\ \rho_{22} - \rho_{11} \end{pmatrix} = \begin{pmatrix} \Omega \\ 0 \\ \Delta \end{pmatrix} \times \begin{pmatrix} \rho_{12} + \rho_{21} \\ -i(\rho_{12} - \rho_{21}) \\ \rho_{22} - \rho_{11} \end{pmatrix} = \begin{pmatrix} i\Delta(\rho_{12} - \rho_{21}) \\ \Delta(\rho_{12} + \rho_{21}) - \Omega(\rho_{22} - \rho_{11}) \\ -i\Omega(\rho_{12} - \rho_{21}) \end{pmatrix} .$$

Hence,

$$\frac{d}{dt} \begin{pmatrix} \rho_{12} \\ \rho_{21} \\ 1 - 2\rho_{11} \end{pmatrix} = \begin{pmatrix} i\Delta\rho_{12} + \frac{i\Omega}{2}(\rho_{11} - \rho_{22}) \\ -i\Delta\rho_{21} - \frac{i\Omega}{2}(\rho_{11} - \rho_{22}) \\ -i\Omega(\rho_{12} - \rho_{21}) \end{pmatrix} .$$