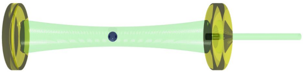
Characterization of

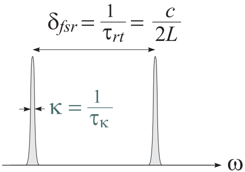
optical cavities

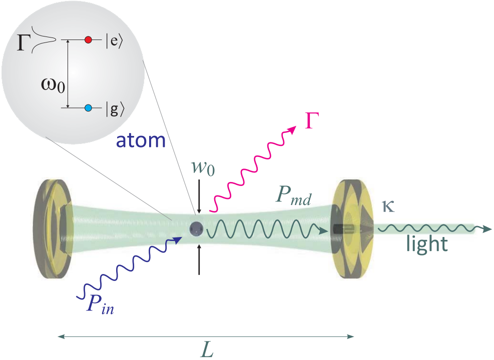


Cavity parameters

Airy formula $\quad I_{\mathrm{trns}} = I_{\mathrm{in}} \frac{1}{1+(2F/\pi)^2 \sin^2(\Delta/2\delta_{\mathrm{fsr}})} \simeq I_{\mathrm{in}} \frac{\kappa^2}{\kappa^2+\Delta^2}$

free spectral range $\quad \delta_{\mathrm{fsr}} = \tau_{\mathrm{rt}}^{-1} = \frac{c}{2L}$, finesse $\quad F = \frac{\pi\delta_{\mathrm{fsr}}}{\kappa} = \frac{\pi\sqrt{R}}{1-R}$



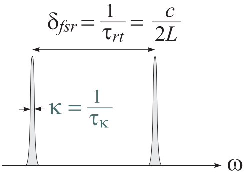


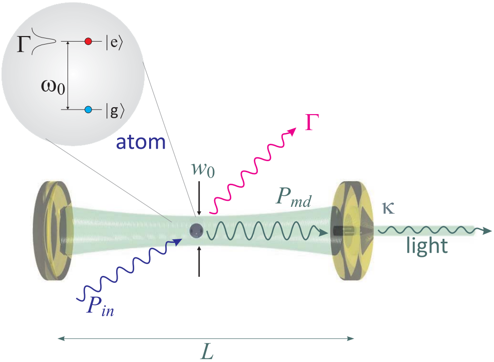
Cavity parameters

Airy formula $$I_{\mathrm{trns}} = I_{\mathrm{in}} \frac{1}{1+(2F/\pi)^2 \sin^2(\Delta/2\delta_{\mathrm{fsr}})} \simeq I_{\mathrm{in}} \frac{\kappa^2}{\kappa^2+\Delta^2}$$

free spectral range $\delta_{\mathrm{fsr}} = \tau_{\mathrm{rt}}^{-1} = \frac{c}{2L}$, finesse $F = \frac{\pi\delta_{\mathrm{fsr}}}{\kappa} = \frac{\pi\sqrt{R}}{1-R}$

radiation field $I(\mathbf{r}) = \frac{2P}{\pi w^2(z)} e^{-2\rho^2/w^2(z)}$ with $\frac{w^2(z)}{w_0^2} = 1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2$





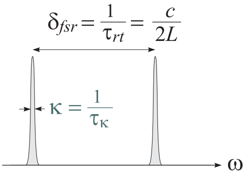
Cavity parameters

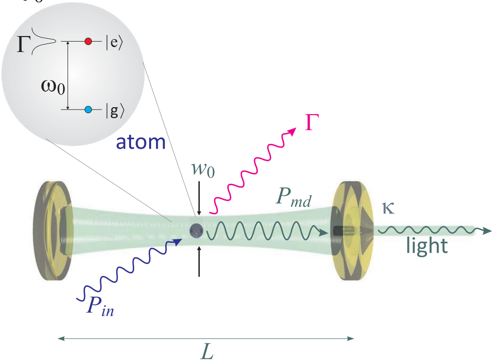
Airy formula $$I_{\text{trns}} = I_{\text{in}} \frac{1}{1+(2F/\pi)^2 \sin^2(\Delta/2\delta_{\text{fsr}})} \simeq I_{\text{in}} \frac{\kappa^2}{\kappa^2+\Delta^2}$$

free spectral range $$\delta_{\text{fsr}} = \tau_{\text{rt}}^{-1} = \frac{c}{2L}$$, finesse $$F = \frac{\pi\delta_{\text{fsr}}}{\kappa} = \frac{\pi\sqrt{R}}{1-R}$$

radiation field $$I(\mathbf{r}) = \frac{2P}{\pi w^2(z)} e^{-2\rho^2/w^2(z)}$$ with $$\frac{w^2(z)}{w_0^2} = 1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2$$

mode volume $$V_{\text{m}} \equiv \frac{1}{I(0)} \int_0^L \int_0^\infty \int_0^{2\pi} I(\mathbf{r}) d\phi \rho d\rho dz = \frac{\pi}{2} L w_0^2$$





Characterization of atom-field interaction

field energy in the *empty* cavity $\quad \frac{\hbar\omega}{2} = \int u_1(\mathbf{r})dV = \frac{1}{c}\int I_1(\mathbf{r})dV = \frac{V_\mathrm{m}}{c} I_1(0) = \frac{V_\mathrm{m}}{c} n\varepsilon_0 c \mathcal{E}_1^2(0)$

Characterization of atom-field interaction

field energy in the *empty* cavity $\frac{\hbar\omega}{2} = \int u_1(\mathbf{r})dV = \frac{1}{c}\int I_1(\mathbf{r})dV = \frac{V_\mathrm{m}}{c} I_1(0) = \frac{V_\mathrm{m}}{c} n\varepsilon_0 c \mathcal{E}_1^2(0)$

field amplitude of a single photon $\mathcal{E}_1 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_\mathrm{m}}}$

Characterization of atom-field interaction

field energy in the *empty* cavity $$\frac{\hbar\omega}{2} = \int u_1(\mathbf{r})dV = \frac{1}{c}\int I_1(\mathbf{r})dV = \frac{V_\mathrm{m}}{c} I_1(0) = \frac{V_\mathrm{m}}{c} n\varepsilon_0 c\mathcal{E}_1^2(0)$$

field amplitude of a single photon $$\mathcal{E}_1 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_\mathrm{m}}}$$

power $$P = \frac{\pi w_0^2 I(0)}{2} = n\hbar\omega\delta_\mathrm{fsr}$$

Characterization of atom-field interaction

field energy in the *empty* cavity $\frac{\hbar\omega}{2} = \int u_1(\mathbf{r})dV = \frac{1}{c}\int I_1(\mathbf{r})dV = \frac{V_{\rm m}}{c} I_1(0) = \frac{V_{\rm m}}{c} n\varepsilon_0 c\mathcal{E}_1^2(0)$

field amplitude of a single photon $\mathcal{E}_1 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{\rm m}}}$

power $P = \frac{\pi w_0^2 I(0)}{2} = n\hbar\omega\delta_{\rm fsr}$

atomic dipole moment $d = \sqrt{\frac{3\pi\varepsilon_0\hbar\Gamma}{k^3}}$

Characterization of atom-field interaction

field energy in the *empty* cavity $\frac{\hbar\omega}{2} = \int u_1(\mathbf{r})dV = \frac{1}{c}\int I_1(\mathbf{r})dV = \frac{V_\mathrm{m}}{c}I_1(0) = \frac{V_\mathrm{m}}{c}n\varepsilon_0 c\mathcal{E}_1^2(0)$

field amplitude of a single photon $\mathcal{E}_1 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_\mathrm{m}}}$

power $P = \frac{\pi w_0^2 I(0)}{2} = n\hbar\omega\delta_\mathrm{fsr}$

atomic dipole moment $d = \sqrt{\frac{3\pi\varepsilon_0\hbar\Gamma}{k^3}}$

$\Longrightarrow$ atom-field coupling force $g \equiv \frac{d\mathcal{E}_1}{\hbar} = \sqrt{\frac{3\pi\Gamma\omega}{2k^3 V_\mathrm{m}}} = \sqrt{\frac{3\Gamma\delta_\mathrm{fsr}}{k^2 w_0^2}}$

Characterization of atom-field interaction

field energy in the *empty* cavity $\frac{\hbar\omega}{2} = \int u_1(\mathbf{r})dV = \frac{1}{c}\int I_1(\mathbf{r})dV = \frac{V_\mathrm{m}}{c} I_1(0) = \frac{V_\mathrm{m}}{c} n\varepsilon_0 c\mathcal{E}_1^2(0)$

field amplitude of a single photon $\mathcal{E}_1 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_\mathrm{m}}}$

power $P = \frac{\pi w_0^2 I(0)}{2} = n\hbar\omega\delta_\mathrm{fsr}$

atomic dipole moment $d = \sqrt{\frac{3\pi\varepsilon_0\hbar\Gamma}{k^3}}$

$\Longrightarrow$ atom-field coupling force $g \equiv \frac{d\mathcal{E}_1}{\hbar} = \sqrt{\frac{3\pi\Gamma\omega}{2k^3 V_\mathrm{m}}} = \sqrt{\frac{3\Gamma\delta_\mathrm{fsr}}{k^2 w_0^2}}$

3 parameters characterizing atom-cavity interaction

Characterization of atom-field interaction

field energy in the *empty* cavity $\frac{\hbar\omega}{2} = \int u_1(\mathbf{r})dV = \frac{1}{c}\int I_1(\mathbf{r})dV = \frac{V_\mathrm{m}}{c}I_1(0) = \frac{V_\mathrm{m}}{c}n\varepsilon_0 c\mathcal{E}_1^2(0)$

field amplitude of a single photon $\mathcal{E}_1 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_\mathrm{m}}}$

power $P = \frac{\pi w_0^2 I(0)}{2} = n\hbar\omega\delta_\mathrm{fsr}$

atomic dipole moment $d = \sqrt{\frac{3\pi\varepsilon_0\hbar\Gamma}{k^3}}$

$\Longrightarrow$ atom-field coupling force $g \equiv \frac{d\mathcal{E}_1}{\hbar} = \sqrt{\frac{3\pi\Gamma\omega}{2k^3 V_\mathrm{m}}} = \sqrt{\frac{3\Gamma\delta_\mathrm{fsr}}{k^2 w_0^2}}$

3 parameters characterizing atom-cavity interaction

- cooperativity Υ
- saturation s
- resolution r

The meaning of cooperativity

$$\beta_0 \quad = \quad \text{single-atom resonant reflection coefficient} \quad = \quad \frac{\text{optical cross section}}{\text{laser beam cross section}} \quad = \quad \frac{\sigma_0}{\pi w_0^2} = \frac{6}{k^2 w_0^2}$$

The meaning of cooperativity

$$\beta_0 \quad = \quad \text{single-atom resonant reflection coefficient} \quad = \quad \frac{\text{optical cross section}}{\text{laser beam cross section}} \quad = \quad \frac{\sigma_0}{\pi w_0^2} = \frac{6}{k^2 w_0^2}$$

$$\Upsilon \quad = \quad \text{cooperativity} \quad \equiv \quad \beta_0 \times \text{ number of round trips} \quad = \quad \beta_0 \frac{F}{\pi} = \frac{6}{k^2 w_0^2} \frac{\delta_{\mathrm{fsr}}}{\kappa} = \frac{2g^2}{\kappa \Gamma}$$

The meaning of cooperativity

$$\beta_0 \quad = \quad \text{single-atom resonant reflection coefficient} \quad = \quad \frac{\text{optical cross section}}{\text{laser beam cross section}} \quad = \quad \frac{\sigma_0}{\pi w_0^2} = \frac{6}{k^2 w_0^2}$$

$$\Upsilon \quad = \quad \text{cooperativity} \quad \equiv \quad \beta_0 \times \text{ number of round trips} \quad = \quad \beta_0 \frac{F}{\pi} = \frac{6}{k^2 w_0^2} \frac{\delta_{\text{fsr}}}{\kappa} = \frac{2g^2}{\kappa \Gamma}$$

$$\Upsilon \quad = \quad \text{cavity-to-free-space scattering ratio} \quad = \quad \frac{\Gamma_{\text{cav}}}{\Gamma_{\text{free}}} \quad = \quad \frac{2F}{\pi} \frac{\Omega_{\text{cav}}}{\Omega_{\text{free}}}$$

with the cavity solid angle $\quad \Omega_{\text{cav}} = \frac{8\pi}{k^2 w_0^2}$

and the dipolar radiation solid angle $\quad \Omega_{\text{free}} = \int_0^{2\pi} \int_0^{\pi} \sin^3 \theta d\theta d\phi \ = \ \frac{3}{8\pi}$

The meaning of cooperativity

β_0 = single-atom resonant reflection coefficient = $\frac{\text{optical cross section}}{\text{laser beam cross section}}$ = $\frac{\sigma_0}{\pi w_0^2} = \frac{6}{k^2 w_0^2}$

Υ = cooperativity $\equiv$ $\beta_0 \times$ number of round trips = $\beta_0 \frac{F}{\pi} = \frac{6}{k^2 w_0^2} \frac{\delta_{\mathrm{fsr}}}{\kappa} = \frac{2g^2}{\kappa\Gamma}$

Υ = cavity-to-free-space scattering ratio = $\frac{\Gamma_{\mathrm{cav}}}{\Gamma_{\mathrm{free}}} = \frac{2F}{\pi} \frac{\Omega_{\mathrm{cav}}}{\Omega_{\mathrm{free}}}$

with the cavity solid angle $\Omega_{\mathrm{cav}} = \frac{8\pi}{k^2 w_0^2}$

and the dipolar radiation solid angle $\Omega_{\mathrm{free}} = \int_0^{2\pi} \int_0^{\pi} \sin^3 \theta d\theta d\phi \; = \; \frac{3}{8\pi}$

critical atom number (# atoms per mode) $N_{\mathrm{crt}} \simeq \frac{1}{\Upsilon}$

$\Upsilon > 1$ $\Longrightarrow$ single atoms make a difference $\Longrightarrow$ strong coupling

The meaning of cooperativity

$$\beta_0 \;=\; \text{single-atom resonant reflection coefficient} \;=\; \frac{\text{optical cross section}}{\text{laser beam cross section}} \;=\; \frac{\sigma_0}{\pi w_0^2} = \frac{6}{k^2 w_0^2}$$

$$\Upsilon \;=\; \text{cooperativity} \;\equiv\; \beta_0 \times \text{ number of round trips} \;=\; \beta_0 \frac{F}{\pi} = \frac{6}{k^2 w_0^2}\frac{\delta_{\text{fsr}}}{\kappa} = \frac{2g^2}{\kappa\Gamma}$$

$$\Upsilon \;=\; \text{cavity-to-free-space scattering ratio} \;=\; \frac{\Gamma_{\text{cav}}}{\Gamma_{\text{free}}} \;=\; \frac{2F}{\pi}\frac{\Omega_{\text{cav}}}{\Omega_{\text{free}}}$$

with the cavity solid angle $\Omega_{\text{cav}} = \frac{8\pi}{k^2 w_0^2}$

and the dipolar radiation solid angle $\Omega_{\text{free}} = \int_0^{2\pi} \int_0^{\pi} \sin^3\theta d\theta d\phi \;=\; \frac{3}{8\pi}$

critical atom number (# atoms per mode) $N_{\text{crt}} \simeq \frac{1}{\Upsilon}$

$\Upsilon > 1 \quad \Longrightarrow \quad$ single atoms make a difference $\quad \Longrightarrow \quad$ strong coupling

collective coupling strength $g_N = g_1\sqrt{N} \quad \Longrightarrow \quad$ strong collective coupling: $N\Upsilon > 1$

The meaning of saturation

single-photon saturation $s = \frac{8g^2}{\Gamma^2}$

critical photon number $n_{\mathrm{sat}} \simeq \frac{1}{s}$

$s > 1$ $\Longrightarrow$ single photon makes a difference $\Longrightarrow$ cavity QED

strong coupling $g \gg \Gamma, \kappa$

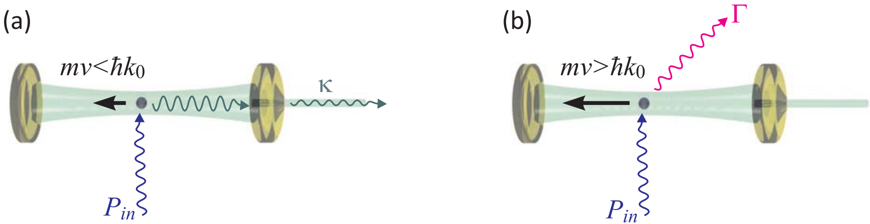
$\Longrightarrow$ resolved vacuum Rabi splitting

$\Longrightarrow$ coherent single-photon Rabi oscillations

The meaning of resolution

single-photon recoil $\quad r \equiv \frac{\omega_{\rm rec}}{\kappa}$

recoil shift $\quad \omega_{\rm rec} \equiv \frac{\hbar k^2}{2m}$



Quantization of relevant degrees of freedom

quantization of matter: $\eta > 1$

quantization of light (strong coupling, CQED): $s > 1$

quantization of motion: $r > 1$

rubidium μ-cavity vs. strontium ring cavity

	rubidium	rubidium (Tübingen)	strontium (SC)	strontium (Paris)
Γ	$6\,\mathrm{MHz}$	$6\,\mathrm{MHz}$	$6.8\,\mathrm{kHz}$	$6.8\,\mathrm{kHz}$
F	250000	250000	250000	48000
κ	$6\,\mathrm{MHz}$	$2.4\,\mathrm{kHz}$	$6\,\mathrm{MHz}$	$1.7\,\mathrm{MHz}$
L	$100\,\mu\mathrm{m}$	$8\,\mathrm{cm}$	$3\,\mathrm{cm}$	$600\,\mu\mathrm{m}$
w_0	$20\,\mu\mathrm{m}$	$70\,\mu\mathrm{m}$	$70\,\mu\mathrm{m}$	$3.5\,\mu\mathrm{m}$
V_0	$0.0002\,\mathrm{mm}^3$	$1\,\mathrm{mm}^3$	$1\,\mathrm{mm}^3$	$6700\,\mu\mathrm{m}^{-3}$
ω_r	$4\,\mathrm{kHz}$	$4\,\mathrm{kHz}$	$5\,\mathrm{kHz}$	$5\,\mathrm{kHz}$
g_1	$18\,\mathrm{MHz}$	$183\,\mathrm{kHz}$	$9\,\mathrm{kHz}$	$1.2\,\mathrm{MHz}$
Υ	9	2.5	$.02$	90
s	9	0.0009379	10	34000
r	0.0008	1.6	0.8	0.003