Bloch equations

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The optical *Bloch* equations describe the interaction of the density matrix with an electromagnetic field over time. In this work, the optical equations of *Bloch* for a two-level atom will be calculated using the density operator. First, we obtain the *Bloch* equations for a two-level atom without taking into account spontaneous emission and we will interpret its solutions. The decay constant will be calculated using the $Weisskopf - Wigner$ theorem which is the usual theory for the study of spontaneous emission. Once the decay constant is obtained, we will calculate the optical equations of *Bloch* with spontaneous emission for a two-level atom, obtain their solutions and analyze the graph of these solutions.

I. INTRODUCTION

In no chemical interaction study, we are interested in examining what happens when a two-year atom interacts with a light source. If we only consider the phenomenon of absorption of a monochromatic light, a Schrödinger equation, which describes a temporal evolution of the state of any system, seriously enough [1]. However, in the face of more complex phenomena, such as simultaneous relaxation and excitation of an atom, a more adequate mathematical formalism that also considers spontaneous emission and any other process in the physical description of the temporal evolution of the interactive temporal phenomenon with a light source will be necessary.

As spontaneous spontaneous emission leaves the atom in a superposition of many states, the wave function that is usually used to describe the situation is replaced by a set of wave functions that only allows obtaining the probability of finding the system in a given state.

In this way, we will use the mathematical formalism of the operator density or density matrix, whose temporal evolution of the elements of this matrix is described by the Bloch equations and not by the Schrödinger equation.

This work is focused on the derivation and interpretation of Bloch's equations.

II. DENSITY OPERATOR

Let's consider a particle enssemble, where a fraction p_1 is in the state $|\psi_1\rangle$, a fraction p_2 is in the state $|\psi_2\rangle$, etc .... a mixed enssemble that cannot be described by a single wave function and for that reason, we will use the density operator for this purpose. The density operator is given by:

$$\hat{\rho} = \sum_i P_i |\psi_i\rangle\langle\psi_i| \tag{1}$$

where $|\psi_i\rangle$ is a complete set of orthonormal states and p_i is the probability of finding the state $|\psi_i\rangle$ in the ensemble. For the case where we have $p_i = 1$, the density operator will be written as:

$$\hat{\rho} = |\psi_i\rangle\langle\psi_i| \tag{2}$$

which sets up a pure enssemble. The elements of the $\hat{\rho}$ array are given by:

$$\langle\psi_i|\hat{\rho}|\psi_i\rangle \tag{3}$$

To obtain the density density matrix we will expand the eigenvectors $|\ \psi_i\rangle$ on an orthonormal basis

$$|\psi_i\rangle = \sum_n C_n^{(i)} |\ n\rangle \tag{4}$$

where we define:

$$C_n^{(i)} = \langle n|\psi_i\rangle \tag{5}$$

By substituting (4) and (5) in (3) and using the completeness relation $\sum_n |n\rangle\langle n| = \mathrm{I}$, we can determine the matrix representation of $\hat{\rho}$ na base $\{|n\rangle\ \}$:

$$\rho = \sum_i \sum_{n,m} P_i C_n^{(i)} C_m^{(i)*} |\ n\rangle\langle\ m| \tag{6}$$

and the matrix elements of the density operator are:

$$\rho_{nm} = \sum_i P_i C_n^{(i)} C_m^{(i)*} = \rho_{mn} \tag{7}$$

where $\hat{\rho}$ is an Hermitian operator.

The diagonal elements are given by:

$$\hat{\rho}_{nm} = \sum_i P_i |C_n^{(i)}|^2 \tag{8}$$

Thus, we have $|C_n^{(i)}|^2$ is a positive number that gives the probability of finding in a measure the state $|n\rangle$ and $\hat{\rho}_{nn}$ represents the average probability of finding the state $|n\rangle$ [2] e. For this reason, $\hat{rho}_{nn}$ is called the state population $|\ n\rangle$.

For elements outside the matrix diagonal,

$$\hat{\rho}_{nm} = \sum_i P_i C_n^{(i)} C_m^{(i)*} \tag{9}$$

where $C_n^{(i)} C_m^{(i)*}$ expresses the effects of interference between the states $\mid n\rangle$ and $\mid m\rangle$ [2] and $\hat{\rho}_{nm}$ elements are called coherence between states. The sum of the diagonal terms is given by the dash that is defined as $Tr[\hat{\rho}] = \sum_n \langle n|\hat{\rho}|n\rangle$ and from the dash we can calculate the expected value of an observable $\langle A\rangle = Tr[\hat{\rho}A]$.

A. Time evolution of density operator

Now, we will express the temporal dependence of the density operator in terms of time-dependent quantum states:

$$|\psi_i(t)\rangle = U(t,t_0)|\psi_i(0)\rangle \tag{10}$$

where the time evolution operator is given by:

$$U(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar} \tag{11}$$

Applying the time evolution operator to the density operator, we can write:

$$\hat{\rho}(t) = U(t,t_0)\hat{\rho}U^{\dagger}(t,t_0) \tag{12}$$

Deriving the two sides of equation (12) in relation to time, we have:

$$\frac{d\hat{\rho}(t)}{dt} = \frac{i}{\hbar}[\hat{\rho}(t), \hat{H}] \tag{13}$$

where equation (13) is called the *Liouville* equation. The *Liouville* equation describes the evolution in time of the density operator, which in turn describes the distribution of a set of quantum states subject to the Hamiltonian $\hat{H}$.

The Hamiltonian $\hat{H}$, can be written in a time dependent part and a time independent part

$$\hat{H}(t) = \hat{H}_0 + \hat{V}(t) \tag{14}$$

where $V(t)$ is the interaction potential of the dipolar transition with the classical oscillating electric field and that describes the time-dependent part, and H_0 is the Hamiltonian that describes the atomic structure. We can write the equation of (13) in the interaction representation, just performing the transformation

$$\tilde{\rho} = e^{i\hat{H}(t-t_0)/\hbar}\hat{\rho}_S(t_0)e^{-i\hat{H}(t-t_0)/\hbar} \tag{15}$$

Repeating the process used to obtain equation (13), we have:

$$\frac{d\tilde{\rho}(t)}{dt} = \frac{i}{\hbar}[\tilde{\rho}(t), \hat{V}(t)] \tag{16}$$

where the temporal evolution of the density operator, in the interaction representation, depends only on the temporal part of the Hamiltonian $\hat{H}$. The interaction representation is recommended for systems whose Hamiltonian has a time dependent and a time independent part.

III. BLOCH EQUATIONS

Now, we will make the deduction of Bloch's optical equations by examining the temporal dependence of the elements of the density operator matrix. So, let's analyze the matrix elements starting with the *Liouville* equation:

$$\begin{aligned}\langle m|\frac{d\hat{\rho}(t)}{dt}|n\rangle &= \frac{i}{\hbar}\langle m|[\hat{\rho}(t),\hat{H}]|n\rangle = \frac{i}{\hbar}\langle m|[\hat{\rho}(t),\hat{H}_0+\hat{V}(t)]|n\rangle \\ &= \frac{i}{\hbar}(E_n-E_m)\langle m|\hat{\rho}(t)|n\rangle + \frac{i}{\hbar}\langle m|[\hat{\rho}(t),\hat{V}(t)]|n\rangle \\ &= \frac{i}{\hbar}(E_n-E_m)\langle m|\hat{\rho}(t)|n\rangle + \frac{i}{\hbar}\sum_k[\langle m|\hat{\rho}(t)|k\rangle\langle k|\hat{V}(t)|n\rangle - \\ &\qquad \langle m|\hat{V}(t)|k\rangle\langle k|\hat{\rho}(t)|n\rangle]\end{aligned} \tag{17}$$

Using the completeness relation $\sum_n |n\rangle\langle n| = \mathrm{I}$ again on the right side of the second term of equation (17), this equation can be rewritten as:

$$\dot{\hat{\rho}}_{nm} = \frac{i}{\hbar}(E_n - E_m)\hat{\rho}_{nm}(t) + \frac{i}{\hbar}\sum_k[\hat{\rho}_{mk}(t)\hat{V}_{kn}(t) - \hat{V}_{mk}(t)\hat{\rho}_{kn}(t)] \tag{18}$$

these are *Bloch* optical equations. The *Bloch* equations are coupled deferential equations that relate the elements of the density operator matrix, in addition to being very useful for describing a two-level system when that system interacts with radiation. In this first approach to the *Bloch* equations, the spontaneous issue term was not included, which will be dealt with later.

IV. BLOCH EQUATIONS FOR TWO LEVEL ATOMS

In this section we will apply the formalism of Bloch's equations to the particle case of the two-level atom coupled to a field without the spontaneous emission term. Considering the fundamental states $|1\rangle$ and excited $|2\rangle$ we will calculate the matrix elements using equation (18). Let's consider the elements outside the diagonal: abel sec: rabi-oscillations

$$\langle 1|\hat{V}|2\rangle = \langle 2|\hat{V}|1\rangle \tag{19}$$

Energy variations will be given by:

$$E_{nm} = E_n - E_m = (n-m)\hbar\omega_0 \tag{20}$$

Thus, we have:

$$\begin{aligned}
\dot{\rho}_{11} &= \frac{i}{\hbar}(\hat{\rho}_{12}\hat{V}_{21} - \hat{V}_{12}\hat{\rho}_{21}) \\
\dot{\rho}_{22} &= \frac{i}{\hbar}(\hat{\rho}_{21}\hat{V}_{12} - \hat{V}_{21}\hat{\rho}_{12}) = -\dot{\rho}_{11} \\
\dot{\rho}_{12} &= i\omega_0\hat{\rho}_{12} + \frac{i}{\hbar}\hat{V}_{12}(\hat{\rho}_{11} - \hat{\rho}_{22}) \\
\dot{\rho}_{21} &= -i\omega_0\hat{\rho}_{21} + \frac{i}{\hbar}\hat{V}_{21}(\hat{\rho}_{22} - \hat{\rho}_{11}) = \dot{\rho}^*_{12}
\end{aligned} \tag{21}$$

remembering that the line must be unitary and the terms outside the diagonal are complex

$$\dot{\rho}_{11} + \dot{\rho}_{22} = 1, \quad \dot{\rho}_{21} = \dot{\rho}^*_{12} \tag{22}$$

Equations (21) are *Bloch* equations written in Schrödinger's representation. These equations can be calculated in the interaction representation Equations (21) are the *Bloch* equations written in Schrödinger's representation. These equations can be calculated on the interaction representation

$$\frac{d\tilde{\rho}_{22}}{dt} = \frac{i}{\hbar}(\tilde{\rho}_{21}\tilde{V}_{12} - \tilde{V}_{21}\tilde{\rho}_{12}), \quad \frac{d\tilde{\rho}_{12}}{dt} = \frac{i}{\hbar}\hat{V}_{12}(\tilde{\rho}_{11} - \tilde{\rho}_{22}) \tag{23}$$

We can also calculate the equations of *Bloch* in the representation of interaction is through substitution $\hat{\rho}_{12} = \tilde{\rho}_{12}e^{i\omega_0 t}$. The interaction representation simplifies coherences by removing their temporal dependency.

The treatment of the system can be simplified by considering the spinning wave approximation (RWA), for which terms that fluctuate rapidly with the frequency $\Delta = \omega + \omega_0$ are neglected, and we only consider the terms that oscillate slowly with frequency $\Delta = \omega - \omega_0$. In this way, we can rewrite the elements of coherence and the potentials of interaction, respectively, as $\hat{\rho}_{12} = \tilde{\rho}_{12}e^{-i\omega t}$, $\hat{\rho}_{12} = \tilde{\rho}_{12}e^{i\omega t}$, $V_{12} = \frac{\hbar}{2}\Omega e^{i\omega t}$ e $V_{21} = \frac{\hbar}{2}\Omega e^{-i\omega t}$ where Ω is the frequency of the light. Making these substitutions in equations (21), we can rewrite the equations of *Bloch*, for a two-level atom, in the matrix form

$$\begin{pmatrix} \dot{\rho}_{11} \\ \dot{\rho}_{22} \\ \dot{\tilde{\rho}}_{12} \\ \dot{\tilde{\rho}}_{21} \end{pmatrix} = \frac{i}{2}\begin{pmatrix} 0 & 0 & \Omega & -\Omega \\ 0 & 0 & -\Omega & \Omega \\ \Omega & -\Omega & -2\Delta & 0 \\ -\Omega & \Omega & 0 & 2\Delta \end{pmatrix}\begin{pmatrix} \rho_{11} \\ \rho_{22} \\ \tilde{\rho}_{12} \\ \tilde{\rho}_{21} \end{pmatrix} \tag{24}$$

To solve this system, we need to diagonalize the 4 x 4 matrix present in equation (24) to calculate its eigenvalues, given by:

$$det(M - \lambda) = \lambda^2(\Delta^2 + \Omega^2) + \lambda^4 = 0 \tag{25}$$

$$\lambda_1 = 0, \quad \lambda_2 = -iG, \quad \lambda_1 = iG \tag{26}$$

where M is the 4 x 4 matrix and $G = \sqrt{\Delta^2 + \Omega^2}$ is the generalized Rabbi frequency. Using *ansatz* $\rho_{ij}(t) = \rho_{ij}(0)e^{\lambda t}$, where λ is the eigenvalue of the M matrix, the solution generated will be given by:

$$\begin{aligned}
\rho_{22}(t) &= \rho^{(1)}_{22}(t) + \rho^{(2)}_{22}(t)e^{iGt} + \rho^{(3)}_{22}(t)e^{-iGt} \\
\tilde{\rho}_{12}(t) &= \tilde{\rho}^{(1)}_{12}(t) + \tilde{\rho}^{(2)}_{12}(t)e^{iGt} + \tilde{\rho}^{(3)}_{12}(t)e^{-iGt}
\end{aligned} \tag{27}$$

Let's assume that the atom is in the ground state when the radiation field is connected at time $t = 0$. Thus, the coefficients $\rho^{(i)}_{22}$ and $\tilde{\rho}^{(i)}_{12}$, which are calculated using the relations of the array elements discussed previously, are:

$$\begin{aligned}
&\rho^{(1)}_{22}(0) = \frac{|\Omega|^2}{2G^2}, \quad \rho^{(2)}_{22}(0) = -\frac{|\Omega|^2}{4G^2}, \quad \rho^{(3)}_{22}(0) = -\frac{|\Omega|^2}{2G^2} \\
&\tilde{\rho}^{(1)}_{12}(0) = \frac{\Omega\Delta}{2G^2}, \quad \tilde{\rho}^{(2)}_{12}(0) = -\frac{(G-\Delta)}{4G^2}, \quad \tilde{\rho}^{(3)}_{12}(0) = \frac{(G+\Delta)}{4G^2}
\end{aligned} \tag{28}$$

Replacing equations (28) in (32) and using the trigonometric properties $cos(x) = 1 - 2sin^2(x/2)$ e $sin(x) = 2sin(x/2)cos(x/2)$, we have the time evolution of the populations, which are given by:

$$\begin{aligned}
\rho_{22}(t) &= \frac{|\Omega|^2}{2G^2}sin^2(\frac{GT}{2}) \\
\rho_{12}(t) &= \frac{|\Omega|^2}{2G^2}sin^2(\frac{GT}{2})[\Delta sin^2(\frac{GT}{2}) + iGcos(\frac{GT}{2})]
\end{aligned} \tag{29}$$

where we already made the transformation $\tilde{\rho}_{12} \longrightarrow \rho_{12}$.

In figure 1, in the graph on the left, we have the time evolution of the population ρ_{22} as a function of time, for fixed values of Ω and Δ. At $t = 0$ the atom is found in the ground state $|2\rangle$. For $t > 0$, we observe population inversions, where the probabilities of finding the atom in the state $|2\rangle$ and fundamental $|1\rangle$ fluctuate over time. The graph on the right in figure 1, the time is kept fixed at $t = \pi/2$ as well as the frequency Ω and we vary the frequency *Delta*. As seen in the graph, for $\Omega = 1$ and $\Omega = 3$ the populations are equally distributed in the fundamental state and in the excited state, in $\Omega = 2$ and $\Omega = 4$ we have the populations fully distributed in the excited and fundamental states, respectively.

V. WEISSKOPF-WIGNER THEORY

The spontaneous emission of an atom in free space is due to fluctuations in the vacuum. Let's consider a

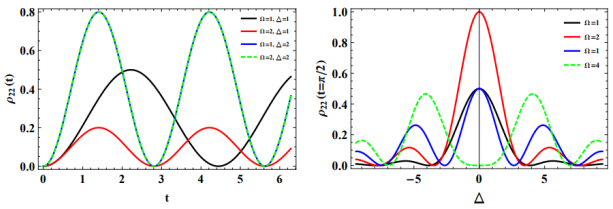

FIG. 1: Time evolution of the population ρ_{22} (left) and as a function of Ω (right).

two-level atom interacting with a finite number of field modes. Each field mode is characterized by its respective $\mathbf{k}$ wave vector. The standard theory of spontaneous emission is the Weisskopf-Wigner [3] theory. We will write the Hamiltonian including the interaction of the atom with the incident radiation and also with the modes of the electromagnetic vacuum.

$$\hat{H} = \hbar g_{\mathbf{k}_0}(\hat{\sigma}^- e^{-i\omega_a t} + \hat{\sigma}^+ e^{i\omega_a t})(\hat{a}^\dagger_{\mathbf{k}0} e^{(i\omega_0 t - i\mathbf{k}_0.\mathbf{r})} + \hat{a}_{\mathbf{k}0} e^{(-i\omega_0 t + i\mathbf{k}_0.\mathbf{r})}) + \sum_{\mathbf{k}} \hbar g_{\mathbf{k}}(\hat{\sigma}^- e^{-i\omega_a t} + \hat{\sigma}^+ e^{i\omega_a t})(\hat{a}^\dagger_{\mathbf{k}} e^{(i\omega_{\mathbf{k}} t - i\mathbf{k}.\mathbf{r})} + \hat{a}_{\mathbf{k}} e^{(-i\omega_{\mathbf{k}} t + i\mathbf{k}.\mathbf{r})}) \tag{30}$$

where $\mathbf{k}_0$, $\omega_{\mathbf{k}_0}$ and $g_{\mathbf{k}_0}$ are the incident radiation waveform, frequency and coupling force, respectively, while $\mathbf{k}$, $\omega_{\mathbf{k}}$ and $g_{\mathbf{k}}$ are, respectively, the wave vector, frequency and coupling force of the electromagnetic vacuum modes. The operators $\hat{\sigma}^-$, $\hat{\sigma}^+$, $\hat{a}^\dagger$ and $\hat{a}$ are operators of, respectively, dexcitation, electron excitation, photon creation and annihilation.

Applying the spinning wave approximation to the Hamiltonian (30), we have:

$$\hat{H} = \frac{\hbar}{2}\Omega_0[\hat{\sigma}^+ \hat{a}^\dagger_0 \mathbf{k}_0 e^{i\Delta_0 t} + h.c.] + \hbar \sum_{\mathbf{k}} [g_{\mathbf{k}} \hat{\sigma} \hat{a}^\dagger_{\mathbf{k}} e^{i\Delta_{\mathbf{k}} t} + h.c.] \tag{31}$$

where we have already introduced the abbreviations $\Delta_0 = \omega_0 - \omega_a$ e $\Delta_{\mathbf{k}} = \omega_{\mathbf{k}} - \omega_a$ e $\Omega_0 = 2\sqrt{n_0} g_{k_0}$ is the frequency of Rabi's interaction.

Since we don't have a stable system, the expanded and written wave function as

$$|\psi(t)\rangle = \alpha(t)|0\rangle_a|0\rangle_{\mathbf{k}} + \beta(t)|1\rangle_a|0\rangle_{\mathbf{k}} + \sum_{\mathbf{k}} \gamma_{\mathbf{k}}(t)|0\rangle_a|1\rangle_{\mathbf{k}} \tag{32}$$

Inserting the Hamiltonian (31) and the wave function (32) in the Schrödinger equation, we obtain the temporal evolution of the amplitudes $\alpha(t)$, $\beta(t)$ and $\gamma_{\mathbf{k}}(t)$

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\frac{i}{\hbar}\hat{H}|\psi(t)\rangle \tag{33}$$

Applying the initial conditions $\alpha(0) = 1$, $\beta(0) = 0$ and $\gamma_{\mathbf{k}}(0) = 0$ to the solutions of equation (33) we obtain the evolution time to $\beta(t)$

$$\dot{\beta}(t) = -i\frac{\Omega_0}{2}\alpha(t)e^{-i\Delta_0 t} - \sum_{\mathbf{k}} g^2_{\mathbf{k}} \int_0^t e^{i\Delta_{\mathbf{k}}(t'-t)}\beta(t')dt' \tag{34}$$

For times $t' << t$, the integrant oscillates very quickly so that there is no significant contribution to the value of the integral. The dominant contribution comes from the times $t' \approx t$. Therefore, we must evaluate $beta(t')$ in the current t and for this reason, $\beta(t')$ must be moved out of the integral. This process, in which atomic decay becomes an out-of-memory process, is called the $Markov$ approximation. Rewriting equation (34), we have:

$$\dot{\beta}(t) = -i\frac{\Omega_0}{2}\alpha(t)e^{-i\Delta_0 t} - \beta(t)\sum_{\mathbf{k}} g^2_{\mathbf{k}} \int_0^t e^{i\Delta_{\mathbf{k}}(t'-t)}dt' \tag{35}$$

Solving equation (35), we have:

$$\dot{\beta}(t) = -i\frac{\Omega_0}{2}\alpha(t) - \frac{\Gamma}{2}\beta(t) \tag{36}$$

where we derive the term Γ which is the decay constant. Therefore, from the $Weisskopf - Wigner$ theory we derive the decay constant Γ that will be used to obtain the equations of $Bloch$ with spontaneous emission. For that, we need to rewrite the equation of (13) with an additional term, which will now be called the master equation.

$$\frac{d\hat{\rho}(t)}{dt} = \frac{i}{\hbar}[\hat{\rho}(t), \hat{H}] + \frac{\Gamma}{2}(2\hat{\sigma}\hat{\rho}\hat{\sigma}^+ - \hat{\sigma}^+\hat{\sigma}\hat{\rho} - \hat{\rho}\hat{\sigma}^+\hat{\sigma}) \tag{37}$$

VI. BLOCH EQUATIONS WITH SPONTANEOUS EMISSION

Now we are in a position to obtain the optical equations of $Bloch$ with the spontaneous emission term. From equations (36) and (37) we can write equations (21) including the decay term

$$\begin{aligned}
\dot{\rho}_{11} &= \frac{i}{\hbar}(\hat{\rho}_{12}\hat{V}_{21} - \hat{V}_{12}\hat{\rho}_{21}) + \Gamma\hat{\rho}_{11} \\
\dot{\rho}_{22} &= \frac{i}{\hbar}(\hat{\rho}_{21}\hat{V}_{12} - \hat{V}_{21}\hat{\rho}_{12}) + \Gamma\hat{\rho}_{22} \\
\dot{\rho}_{12} &= i\omega_0\hat{\rho}_{12} + \frac{i}{\hbar}\hat{V}_{12}(\hat{\rho}_{11} - \hat{\rho}_{22}) + \Gamma\hat{\rho}_{12} \\
\dot{\rho}_{21} &= -i\omega_0\hat{\rho}_{21} + \frac{i}{\hbar}\hat{V}_{21}(\hat{\rho}_{22} - \hat{\rho}_{11}) + \Gamma\hat{\rho}_{21}
\end{aligned} \tag{38}$$

Equations (38) can be written in matrix form

$$\begin{pmatrix} \dot{\rho}_{11} \\ \dot{\rho}_{22} \\ \dot{\tilde{\rho}}_{12} \\ \dot{\tilde{\rho}}_{21} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2\Gamma & 0 & i\Omega & -i\Omega \\ 2\Gamma & 0 & -i\Omega & i\Omega \\ i\Omega & -i\Omega & -2(i\Delta - \lambda) & 0 \\ -i\Omega & i\Omega & 0 & 2(i\Delta - \lambda) \end{pmatrix}\begin{pmatrix} \rho_{11} \\ \rho_{22} \\ \tilde{\rho}_{12} \\ \tilde{\rho}_{21} \end{pmatrix} \tag{39}$$

The spontaneous emission introduced by the Γ decay constant allows the system to reach a steady state. Stationary solutions can be obtained by making time derivatives $\dot{\rho}_{ij}$ equal to zero. Thus, we have:

$$\begin{aligned}\rho_{22}(0) &= \frac{1}{4}\frac{|\Omega|^2}{\Delta^2 + \frac{1}{2}|\Omega|^2 + \frac{\Gamma^2}{4}} \\ \rho_{12}(0) &= \frac{e^{i\Delta t}}{2}\frac{\Omega(\Delta - i\Gamma/2)}{\Delta^2 + \frac{1}{2}|\Omega|^2 + \frac{\Gamma^2}{4}}\end{aligned} \tag{40}$$

as populations and coherences depend on the frequency Ω and the decay constant Γ, we can define an effective width, given by:

$$\Gamma_{ef} = 2\sqrt{\frac{1}{2}|\Omega|^2 + \frac{\Gamma^2}{4}} \tag{41}$$

The decay constant can also be written on $\Gamma = 2\gamma$.

The behavior of the ρ_{22} porpulation as a function of γ is shown in figure-2. As we increase the γ decay constant, the graph line widens. The effective width Γ_{ef} depends on the frequency which in turn depends on the applied electric field $\Omega = d_{12}.\mathbf{E}_0/\hbar$. This whole process is called power extension.

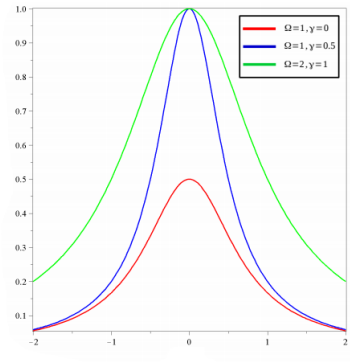

FIG. 2: Population graph ρ_{22} (vertical axis) as a function of the frequency Δ (horizontal axis) in steady state.

VII. CONCLUSION

In this work, we determine Bloch's optical equations from the density operator. To calculate the Bloch equations, we obtain the temporal evolution of the elements of the matrix of the density operator in a $\{|n\rangle\}$basis .

Soon after, we calculate the Bloch equations for the particular case of a two-level atom, without the dissipation term, and conclude that populations oscillate their probabilities of being found in fundamental states and excited over time, to fixed values of Ω and Δ. Keeping the time fixed at $t = \pi/2$ as well as the frequency Ω and we vary the frequency Δ, noting that the populations can be equally distributed between fundamental and exited states or be fully distributed or in ground state or excited state, depending on the values of Ω.

The decay term, responsible for spontaneous emission, was determined using the $Weisskopf - Wigner$ theory, from which we obtain the master equation. From the master equation, it was possible to calculate Bloch's equations with spontaneous emission for a two-level atom and from them, we obtain stationary solutions for populations and for coherences. From the population graph, we observe that the width of the curve depends on the decay term, whereas the *widening by power* depends on the applied electric field.

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